A Root-Mean-Square-based Measurement Method to Optimize a Parameter in the Control Systems Design

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***Abstract*—In the classical control systems design, a parameter is typically determined by the system's time response or frequency response. A standard unstable double-integrator plant with a spring constant K and a mass M is a perfect illustrative example. A stabilizing damper B is determined by setting the time-response to be qualitatively fast and non-oscillatory, which is basically setting the damping-ratio of the second order system equal to 1. The time-response is the system's response to a certain input, such as an impulse-function, a step-function, etc., not its response to an arbitrary disturbance, which is more likely to occur in the field The root-mean-square-(rms)-based approach introduced here allows a designer to estimate a better parameter to minimize an objective function. The objective function is a combined quadratic function of the "error" and the "effort" rms values, both to be minimized by selecting the damper B accurately for each of the 3 (three) different schemes of disturbances. After successfully applied to the standard double-integrator plant given as an illustrative example, the similar approach is implemented for an armature-controlled dc motor speed-control design.**

***Keywords*—root mean square; rms; objective function; double-integrator, disturbance, dc motor**

I. INTRODUCTION

Classical methods to determine a parameter in the control systems design have been developed for almost a hundred years [1] since the steam engine's governor was invented. The most common methods are derived analytically from the system's responses to a certain input. The time response of a system to a step-input is one of the most common ways to characterize the performance of a control system [2]. The frequency response is also commonly used such as in methods based on the Nyquist criteria [3], or other methods.

Those methods based on the time response or the frequency response or both are usually valid and analytically verified, but in the field they - at least most of them - are not easily implemented due to a couple of reasons, among others for instance: (1) the methods require the use of sophisticated equipment such as an oscilloscope with the capability of displaying one-shot signals or a spectrum-analyzer for the frequency response, (2) the inputs should be a certain kind that is not easy to generate (an "ideal" step-function or an impulse-function $\delta(t)$ never actually exists), (3) even if the specific inputs may be approximated for some cases, they are not always applicable, for example: a step input of armature voltage cannot be actually applied to a large DC motor.

Measuring equipments - particularly used in electrical engineering - mostly display the measurement results in rms values - also called "effective" values - especially when the signal being measured varies with time considerably [4]. Other physical units are typically converted into electrical quantities using sensors, and then their rms values are measured.

II. THE BASIC CONCEPT

This method is proposed to be applied in a practical situation in the field, so that all quantities are supposed to be physically - or to be more specific: electrically - measureable. All calculations involving the measured quantities should be made available either in analog or digital forms, or both, at the real time during the system operation. The following defined variables are in scalar form for single-input single-output systems, however, they can be generalized into vector and matrix forms for multi-input multi-output systems or systems with multi-variables.

Firstly, we define the *error* e(t) as the deviation of the actual output y(t) as compared to the desired output y'(t):

$$e(t)=y(t)-y'(t) \tag{1}$$

This error is caused by an arbitrary disturbance, which is supposed to be repeatable, most preferably periodic, so that we can measure the rms value of e(t), E, as follows [5]:

$$E=\sqrt{\frac{1}{T}\int_0^T [e(t)]^2 dt} \tag{2}$$

where T is the time period of the disturbance if it is periodic. If the disturbance is not periodic, then we determine T as a finite period of time when the disturbance still considerably affecting the output.

Secondly, we define the control *effort* u(t) as a designed variable to reduce the rms error E:

$$u(t)=f(c_i \in \mathbf{C}, e(t)) \tag{3}$$

where $\mathbf{C} = \{c_1, c_2, c_3, ..., c_n\}$ is a set of control parameters to optimize the system and f(*) is the control algorithm.

The rms value control effort u(t), U, is measured as follows:

$$U=\sqrt{\frac{1}{T}\int_0^T [u(t)]^2 dt} \tag{4}$$

A control system is said to be "correctly designed" if increasing the *effort* is effectively decreasing the *error*, and vice versa. This characteristic is very important to guarantee the existence of optimum design obtained by minimizing an objective function J, subject to the variations of control parameters $\mathbf{C} = \{c_1, c_2, c_3, ..., c_n\}$ formulated as the following standard unconstrained optimization problem [6]:

$$\text{minimize:} \quad J=\sqrt{[pE]^2+[qU]^2} \tag{5}$$

subject to the variations of $\mathbf{C} = \{c_1, c_2, c_3, ..., c_n\}$

where p and q are coeficients to give a freedom to the designer to emphasize the weight of either the error or the effort, or both, of their contributions to the objective function J. Those coeficients are also useful to equalize the physical units of E and U if they are obtained from different measurement systems. The square-root is optional, just in case the numerical results of squaring the quantities are either too large or too small.

A well established case of control design in the classical control theory is the unstable double-integrator plant model of a spring-mass-damper system with a mass **M** connected to a reference wall by a spring with the constant **K** stabilized by a damper with the constant **B** as represented by the block-diagram shown in Fig. 1. We use this case as an illustrative example of the basic concept described previously.

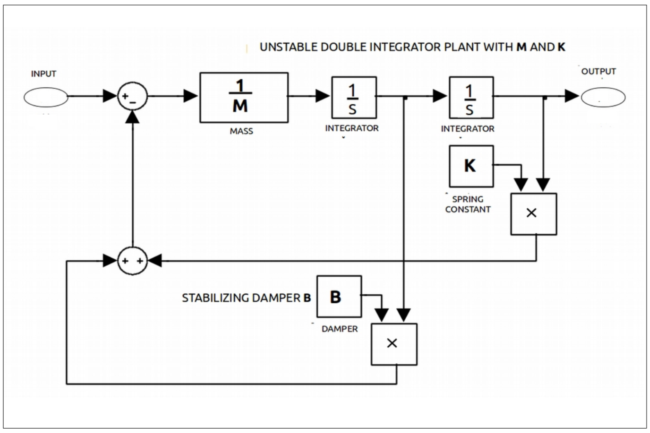

Fig. 1 An unstable double-integrator plant with a mass M and a spring constant K, stabilized by a damper B

The most common time-response is characterized by a step input. For a large **B**, the response is over-damped, which is characterized as too slow, while for a small **B**, the response is under-damped, characterized as too oscillatory. The “best” (both fast and non-oscillatory) performance is believed to be attained by setting the damping ratio **ξ** = 1, or:

$$\mathbf{B}=2\sqrt{\mathbf{MK}} \tag{6}$$

The response for **ξ** = 1 is also characterized as critically damped response. A “normalized” plant with **M** = 1 unit mass and **K** = 1 unit spring constant requires **B = 2** units of damping constant to exhibit a critically damped response to a step input, as shown Fig. 2.

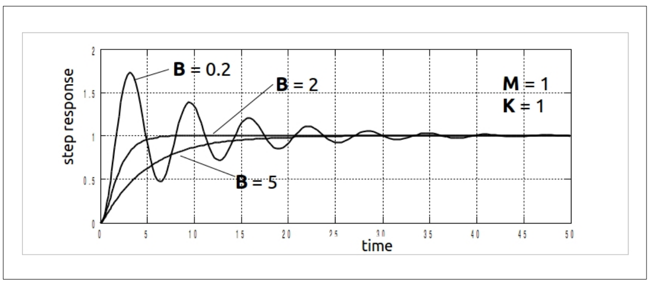

Fig. 2 The step-responses for **B** = 2 (*critically damped*), for **B** = 5 (*over-damped*) and for **B** = 0.2 (*under-damped*)

The desired output y'(t) of this system is 0, so by using (1) we define the error e(t) as the actual output y(t), and the rms value of the error e(t) is also the rms value of the actual output y(t) measured using (2):

$$E=\sqrt{\frac{1}{T}\int_0^T [y(t)]^2\,dt} \tag{7}$$

The control effort is defined as the force caused by the damper, then by using (3), the control algorithm can be derived as:

$$u(t)=\mathbf{B}\frac{dy(t)}{dt} \tag{8}$$

and using (4), the rms value of the control effort is determined as:

$$U=\sqrt{\frac{1}{T}\int_0^T [\mathbf{B}\frac{dy(t)}{dt}]^2\,dt} \tag{9}$$

Finally, by setting up the coefficients p = q = 1, the objective function J is derived using (5) as follows:

$$J=\sqrt{\frac{1}{T}[\int_0^T [[y(t)]^2+[\mathbf{B}\frac{dy(t)}{dt}]^2]\,dt]} \tag{10}$$

J is to be minimized subject to the variation of **B**.

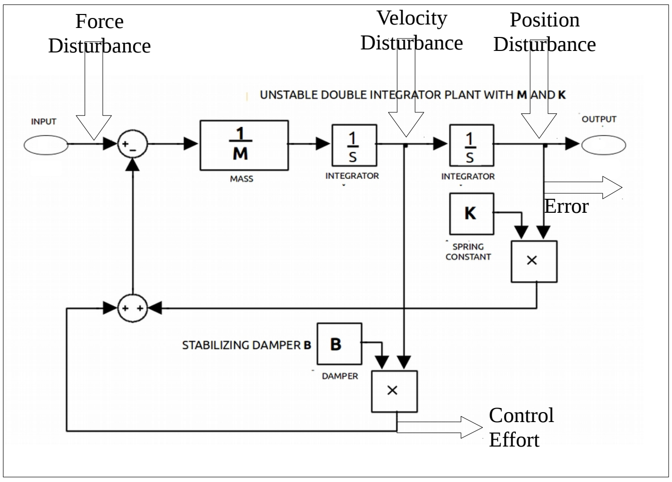

Fig. 3 Force, Velocity and Position Disturbances, Error and Control Effort

An arbitrary disturbance x(t) can be applied as one of three types:

(1) *Force Disturbance*, where the actual output y(t) is the solution of a differential equation:

$$\mathbf{M}\frac{d^2y(t)}{dt^2}+\mathbf{B}\frac{dy(t)}{dt}+\mathbf{K}\,y(t)=x(t) \tag{11}$$

(2) *Velocity Disturbance*, where the actual output y(t) is the solution of a differential equation:

$$\mathbf{M}\frac{d^2y(t)}{dt^2}+\mathbf{B}\frac{dy(t)}{dt}+\mathbf{K}\,y(t)=\mathbf{M}\frac{dx(t)}{dt} \tag{12}$$

(3) *Position Disturbance*, where the actual output y(t) is the solution of a differential equation:

$$\mathbf{M}\frac{d^2y(t)}{dt^2}+\mathbf{B}\frac{dy(t)}{dt}+\mathbf{K}\,y(t)=\mathbf{M}\frac{d^2x(t)}{dt^2}+\mathbf{B}\frac{dx(t)}{dt} \tag{13}$$

Without control effort u(t) or **B** = 0, and after a time period of T when the disturbunce x(t) is not considerably affecting the actual output y(t), the actual output y(t) approximates the solution of the differential equation:

$$\mathbf{M}\frac{d^2 y(t)}{dt^2}+\mathbf{K}\,y(t)=0 \qquad (14)$$

which is a sustained oscillation:

$$y(t)=\sqrt{(\frac{\boldsymbol{M}}{\boldsymbol{K}})}\,\boldsymbol{A}\sin(\sqrt{(\frac{\boldsymbol{K}}{\boldsymbol{M}})}\,t) \qquad (15)$$

For a "normalized" plant with **M** = 1 unit mass and **K** = 1 unit spring constant, the actual output y(t) is a sinusoidal signal with the angular frequency $\boldsymbol{\omega}$ = 1 rad/sec and with the amplitude **A** depending upon the magnitude and the duration of the disturbances:

$$y(t)=\boldsymbol{A}\sin(t) \qquad (16)$$

whose rms value is known as $\frac{\boldsymbol{A}}{\sqrt{2}}$ or 0.7071068**A**. Thus, for **B**=0, the rms value of the actual output y(t), the rms value of error e(t) and the objective function J is approaching 0.7071068**A** for all three types of disturbance. Fig. 4 shows all actual outputs of the three types of disturbance and the measurement of their rms values.

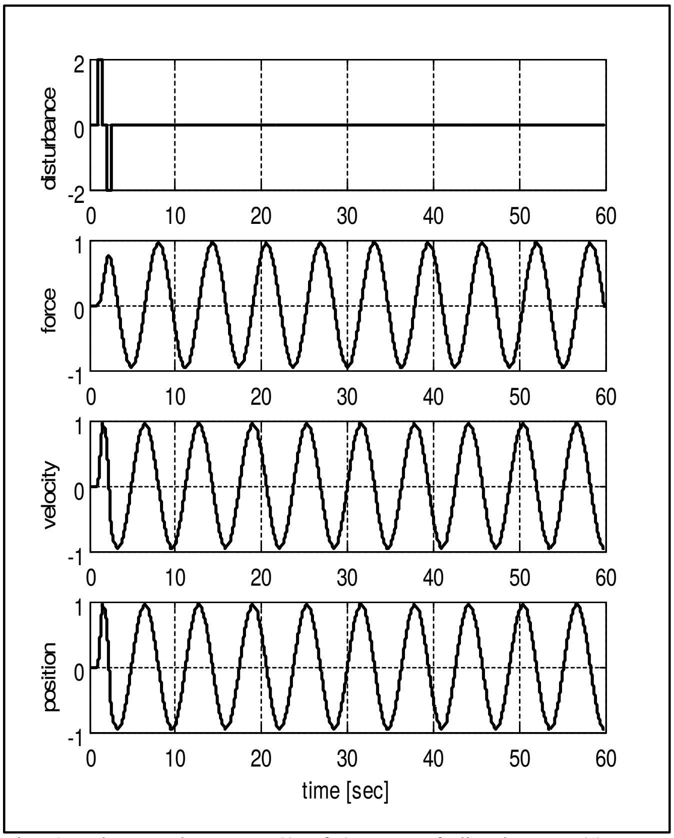

Fig. 4 The actual output y(t) of 3 types of disturbances: (1) *Force Disturbance*, (2) *Velocity Disturbance* and (3) *Position Disturbance*, all for no control effort or **B** = 0.

The application of **B** ≥ 0 for all three types of disturbances indicates that this control system is "correctly designed" because increasing the control effort is effectively decreasing the error as clearly shown in Fig. 5, Fig. 6 and Fig. 7. This guarantees the existence of optimum **B** for each case, as described in Table 1 for an objective function J with coefficients p = q = 1. Fig. 8 shows the actual output of each type of disturbance when an optimum **B** for the objective function J with p = q = 1 is applied.

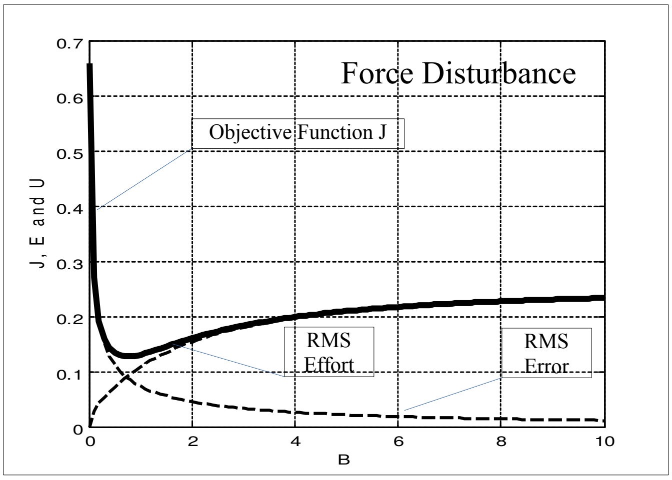

Fig. 5 The Error, Control Effort and the Objective Function (with coefficients p = q = 1) of the Force Disturbance for Various Constant Spring B

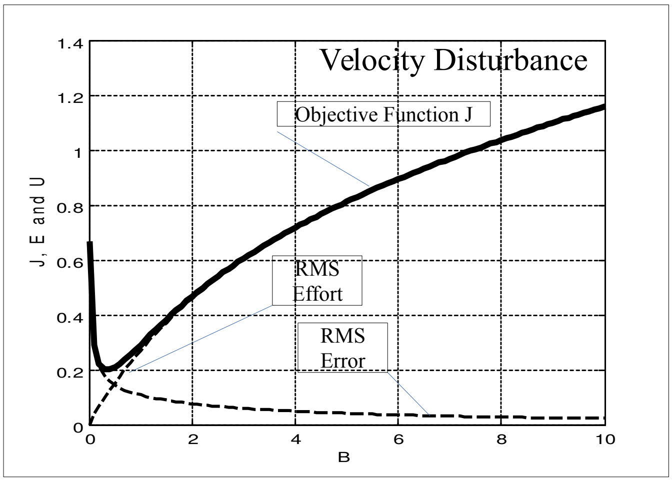

Fig. 6 The Error, Control Effort and the Objective Function (with coefficients p = q = 1) of the Ve;ocity Disturbance for Various Constant Spring B

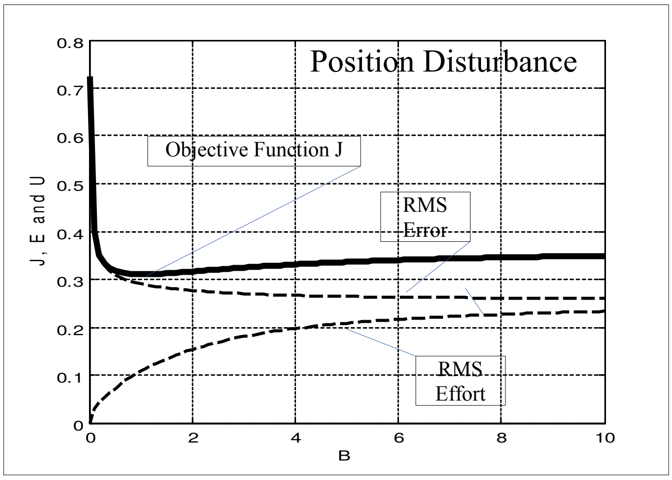

Fig. 7 The Error, Control Effort and the Objective Function (with coefficients p = q = 1) of the Position Disturbance for Various Constant Spring B

Table 1 The Range of Minimizing B and the Damping Ratio ξ for each Type of Disturbance

Disturbance Type	Range of Minimizing B	Minimizing ξ (by curve fitting)	Characteristics
Force	0.6 - 0.8	0.3739	underdamped
Velocity	0.3 - 0.5	0.1847	underdamped
Position	0.9 - 1.1	0.5184	underdamped

The actual output for optimum **B**, with coefficients of the objective function J equally emphasizing the error dan the control effort (p = q = 1), as shown in Fig. 8, looks too oscillatory, because the damping ratio ξ of these configurations are all less than **1** (underdamped).

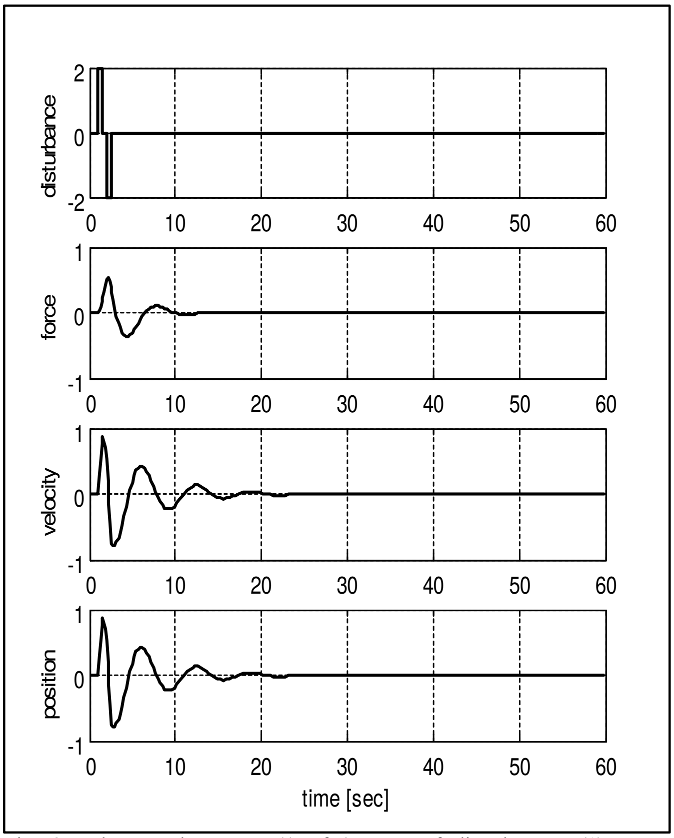

Fig. 8 The actual output y(t) of 3 types of disturbances: (1) *Force Disturbance* with **B**=0.7478, (2) *Velocity Disturbance* with **B**=0.3594 and (3) *Position Disturbance* with **B**=1.0368

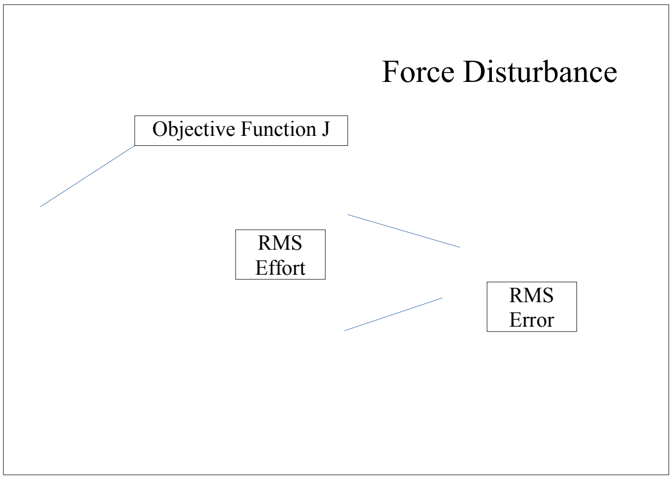

Fig. 9 The Error, Control Effort and the Objective Function (with coefficients p = ... and q = 1) of the Force Disturbance for Various Constant Spring B

The oscillatory behavior of the actual output can be reduced by increasing the system's damping ratio ξ, or - in this case - the damper constant **B**. Greater **B** optimum can be obtained by putting more emphasize on the rms value of the error E in the objective function J, or setting up p > q. For example, the curves of J, E and U looks like the graph shown in Fig. 9 for p = and q = 1. The optimum **B** is 2, the same as suggested by the classical control analysis (see Eq. (6) previously described), and the actual output is shown in Fig. 10, which is much less oscillatory than before. Thus, this method gives more quantitative and solid basis to the reasoning on the selection of critically damped response for determination of the damper constant **B**.

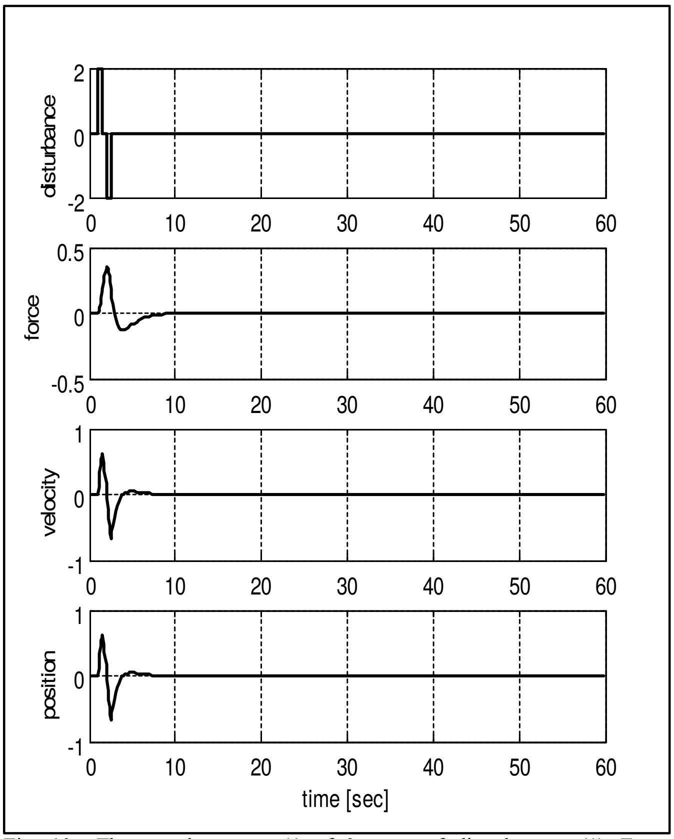

Fig. 10 The actual output y(t) of 3 types of disturbances: (1) *Force Disturbance,* (2) *Velocity Disturbance* and (3) *Position Disturbance all* with **B**=2

III. The Practical Application

The idea of developing this measurement method came up for the first time when an armature-controlled dc motor required a PID controller to maintain its constant speed during its operation in a changing load condition [7]. The problem of selecting the "best" controller is practically solved by applying this method.

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- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as “3.5-inch disk drive.”
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(1)

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- In American English, commas, semi-/colons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
- A graph within a graph is an “inset,” not an “insert.” The word alternatively is preferred to the word “alternately” (unless you really mean something that alternates).
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- Do not confuse “imply” and “infer.”
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- The abbreviation “i.e.” means “that is,” and the abbreviation “e.g.” means “for example.”

An excellent style manual for science writers is [7].

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TABLE I. Table Styles

Table Head	**Table Column Head**		
	Table column subhead	***Subhead***	***Subhead***
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a. Sample of a Table footnote. *(Table footnote)*

b.

Fig. 1. Example of a figure caption. *(figure caption)*

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity "Magnetization," or "Magnetization, M," not just "M." If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write "Magnetization (A/m)" or "Magnetization (A (m(1)," not just "A/m." Do not label axes with a ratio of quantities and units. For example, write "Temperature (K)," not "Temperature/K."

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References

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Capitalize only the first word in a paper title, except for proper nouns and element symbols.

For papers published in translation journals, please give the English citation first, followed by the original foreign-language citation [6].

REFERENCES

[1] G. Eason, B. Noble, and I.N. Sneddon, “On certain integrals of Lipschitz-Hankel type involving products of Bessel functions,” Phil. Trans. Roy. Soc. London, vol. A247, pp. 529-551, April 1955. (*references*)

[2] J. Clerk Maxwell, A Treatise on Electricity and Magnetism, 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68-73.

[3] I.S. Jacobs and C.P. Bean, “Fine particles, thin films and exchange anisotropy,” in Magnetism, vol. III, G.T. Rado and H. Suhl, Eds. New York: Academic, 1963, pp. 271-350.

[4] K. Elissa, “Title of paper if known,” unpublished.

[5] R. Nicole, “Title of paper with only first word capitalized,” J. Name Stand. Abbrev., in press.

[6] Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, “Electron spectroscopy studies on magneto-optical media and plastic substrate interface,” IEEE Transl. J. Magn. Japan, vol. 2, pp. 740-741, August 1987 [Digests 9th Annual Conf. Magnetics Japan, p. 301, 1982].

[7] M. Young, The Technical Writer’s Handbook. Mill Valley, CA: University Science, 1989.