* Model Ruang Keadaan (State Space)

- ⇒ MIMO
- ⇒ "time domain" → f(t)
- ⇒ MATRIX ⇝ simulasi Komputer
- ⇒ "internal" → "state variable" (peubah keadaan)
- ⇒ "general"

Konfigurasi umum

$u \Rightarrow \boxed{\begin{matrix}\dot{x} = Ax + Bu \\ y = Cx + Du\end{matrix}} \Rightarrow y$

Persamaan Keadaan / State Equation: $\dot{x} = Ax + Bu$

Persamaan Keluaran / Output Equation: $y = Cx + Du$

$u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$: vektor $[m \times 1]$ m buah masukan

$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix}$: vektor $[k \times 1]$ k buah keluaran

$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$: vektor $[n \times 1]$ n buah peubah keadaan

$\dot{x} = \frac{dx(t)}{dt} = \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \vdots \\ \frac{dx_n(t)}{dt} \end{bmatrix}$

$\underset{[n \times 1]}{\dot{x}} = \underset{[n \times n]}{A}\,\underset{[n \times 1]}{x} + \underset{[n \times m]}{B}\,\underset{[m \times 1]}{u}$, $[n\times1]$

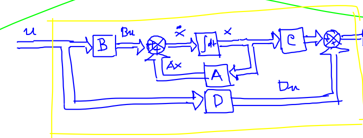
$A\,[n \times n]$ $B\,[n \times m]$

$\underset{[k \times 1]}{y} = \underset{[k \times n]}{C}\,\underset{[n\times 1]}{x} + \underset{[k \times m]}{D}\,\underset{[m \times 1]}{u}$, $[k\times1]$

$C\,[k \times n]$ $D\,[k \times m]$

* Bagan Kotak :

$u \Rightarrow \boxed{\begin{matrix}\dot{x} = Ax + Bu \\ y = Cx + Du\end{matrix}} \Rightarrow y$



* Analisis Keadaan tunak (steady state)

$x(t)$ = isyarat periodik → $x(kT) = x(T)$, $k = 1, 2, 3, \ldots$

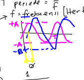
T periode $= \frac{1}{f}$

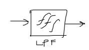
f = frekuensi [Hertz]

$x(t) = A \sin \omega t$

$\omega = 2\pi f$ "omega"

$s \rightarrow j\omega$ $j = \sqrt{-1}$





$X(s) = \mathcal{L}\,x(t) = X(j\omega)$

$G(s) = \frac{1}{RCs+1} \longrightarrow G(j\omega) = \frac{1}{j\omega RC + 1}$

$y(t) = B \sin(\omega t + \phi)$

→ Gain (penguatan) BATI $= \frac{B}{A} = \left|\frac{1}{j\omega RC+1}\right| = \frac{1}{\sqrt{\omega^2R^2C^2+1}}$ → Low Pass Filter

$\hookrightarrow |G(j\omega)|$

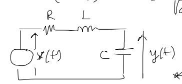
→ Phase Shift (pergeseran fasa) $= \phi = \angle G(j\omega) = -\tan^{-1}(\omega RC)$

$f \to 0 \Rightarrow \omega \to 0$ $\phi \Rightarrow -\tan^{-1} 0 = 0$

$f \to \infty \Rightarrow \omega \to \infty$ $\phi \Rightarrow -\tan^{-1} \infty = -90^\circ$

$y(t) = B \sin(\omega t + \phi) \longrightarrow B = \frac{A}{\sqrt{\omega^2R^2C^2+1}}, \ \phi = -\tan^{-1}(\omega RC)$

Latihan :



$X(s) \rightarrow \boxed{G(s)} \rightarrow Y(s)$

* Analisis Transient

$x(t) = V\,u(t) \longrightarrow y(t) = \ldots ?$ $t \geq 0$

* Analisis Steady state

$x(t) = A \sin \omega t \longrightarrow y(t) = B \sin(\omega t + \phi)$

$B = \ldots ?$

$\phi = \ldots ?$

next : Pemodelan Ruang Keadaan (State Space)
BA'DA DZUHUR !

Contoh : Sebuah sistem dengan 3 input, 4 output dan 5 peubah keadaan dimodelkan dengan Model Ruang Keadaan. Tentukan dimensi matrix A, B, C dan D.

Jawab : $m = 3$ $n = 5$ $k = 4$

$A\,[n \times n] \longrightarrow A\,[5 \times 5]$

$B\,[n \times m] \longrightarrow B\,[5 \times 3]$

$C\,[k \times n] \longrightarrow C\,[4 \times 5]$

$D\,[k \times m] \longrightarrow D\,[4 \times 3]$

Catatan. * Semua matrix A, B, C dan D berupa konstanta → sistem LTI (Linear Time Invariant)

* Ada di antara matrix A, B, C dan/atau D berupa matrix fungsi waktu t → sistem LTV (Linear Time Varying)

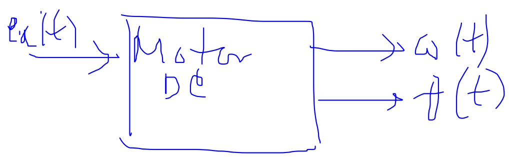
* selain di atas, sistem tak linier

mis. : $A = \begin{bmatrix} x_1 & u_1 \\ u_2 & x_2 \end{bmatrix}$

$\begin{bmatrix} 1 & K & M \\ 10 & 25 & B \\ & \ldots & \end{bmatrix}$ ✓

$\begin{bmatrix} \sin t & e^{-t} & f(t) \end{bmatrix}$

Contoh :



1 input $m = 1$

2 output $k = 2$

3 peubah keadaan $n = 3$

$A\,[3 \times 3]$, $B\,[3 \times 1]$, $C\,[2 \times 3]$, $D\,[2 \times 1]$ ⇐ ✓

"State Assignment" : Misalnya : $u \triangleq e_a(t)$

$y_1 \triangleq \omega(t)$ $y_2 \triangleq \theta(t)$

$x_1 \triangleq i_a(t)$, $x_2 \triangleq \omega(t)$, $x_3 \triangleq \theta(t)$

$\frac{di_a(t)}{dt} = \frac{1}{L}\left[e_a(t) - K_b\,\omega(t) - R\,i_a(t)\right]$

$\frac{d\omega(t)}{dt} = \frac{1}{J}\left[K_m\,i_a(t) - B\,\omega(t)\right]$

$\frac{d\theta(t)}{dt} = \omega(t)$