Numerical ——

Pemodelan & Metode Numerik

Pemodelan NUMERIK → RHz
Metode NUMERIK → Dr. Amil

website:

www.unhas.ac.id/rhiza/arsip/kuliah/

Penilaian:

$$NA = \frac{\text{Nilai RHz} + \text{Nilai AMIL}}{2}$$

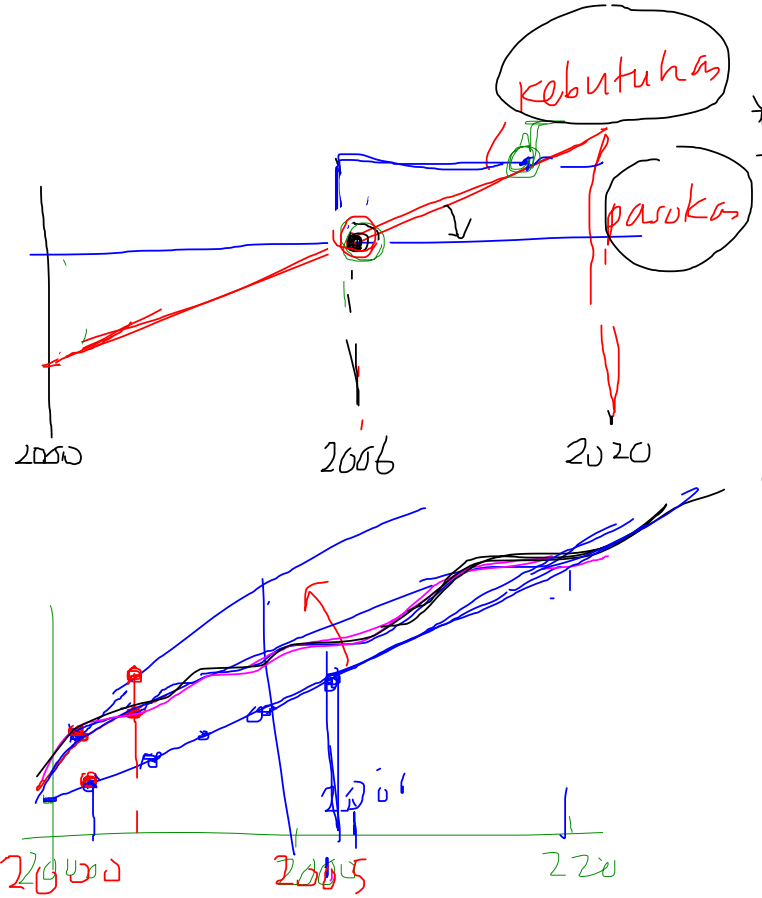
(0 - 100) ↓

0 - 100 → A, A-, B+, B ..... dst
≥ 86 ——

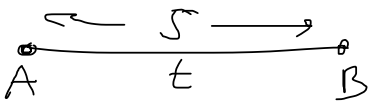
→ Nilai RHz ↔ Tugas 1, 2, 3
Ujian FINAL

rhiza@unhas.ac.id
#6281643121 62

} 24 jam



Time Continuum



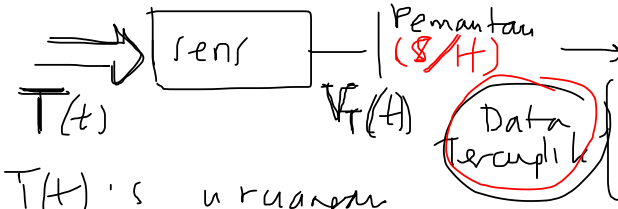
Kecepatan : $\frac{S}{t}$

$$v(t) = \frac{dS}{dt}$$

ANALOG

Kontinyu

Discrete_Time



T(t) : suhu ruangan

Data tercuplik

report

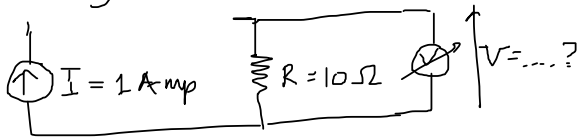
Waktu	Suhu
01.00	25
02.00	23
03.00	26

Discrete Time

DIGITAL

Contoh Sistem Deterministik & Stochastik

* Rangkaian Listrik

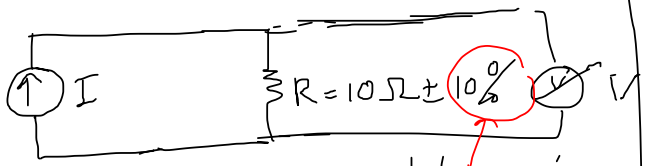


Hukum ohm $V = I.R$.

$I = 1$ Amp → $V = 1 \times 10 = 10$ Volt

$I = 2$ Amp → $V = 2 \times 10 = 20$ Volt

<DETERMINISTIK>



$9\Omega \le R \le 11\Omega \rightarrow 10\%$

$I = 1$ Amp → $9 \text{ Volt} \le V \le 11$ Volt

$I = 2$ Amp → $18 \text{ Volt} \le V \le 22$ Volt

<STOKHASTIK>

VISIBILITAS

"stokhastik"

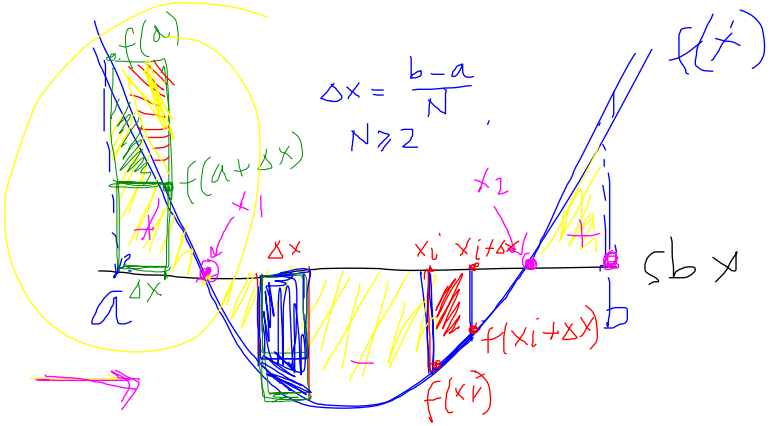
"deterministik"

RU'YAT

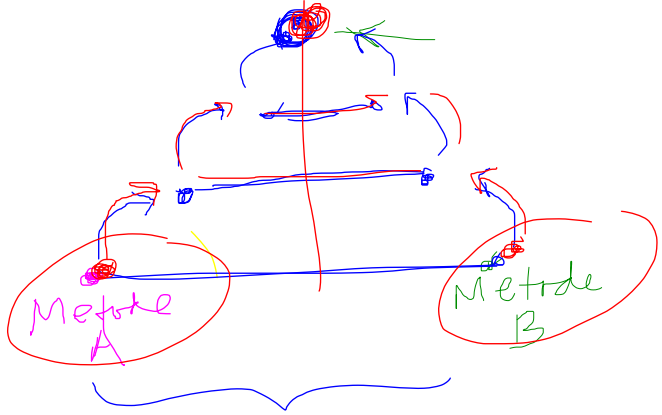
HISAB

POSISI

Imkanur Ru'yat

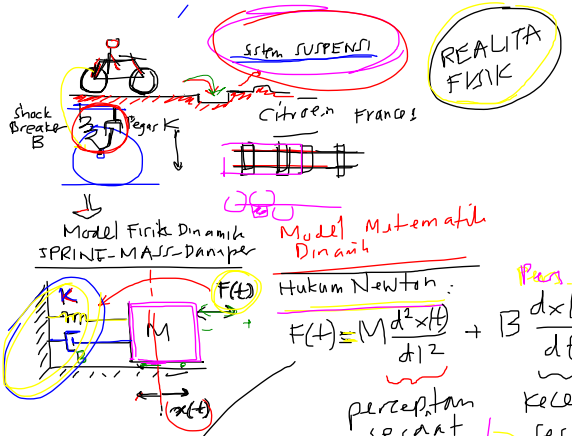


$$\Delta x = \frac{b-a}{N}, \quad N \geq 2$$



estimasi error

metode trapesium lebih bagus daripada metode 4-percep pangi



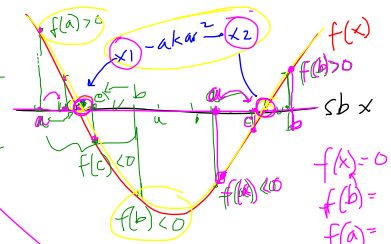
Hukum Newton :

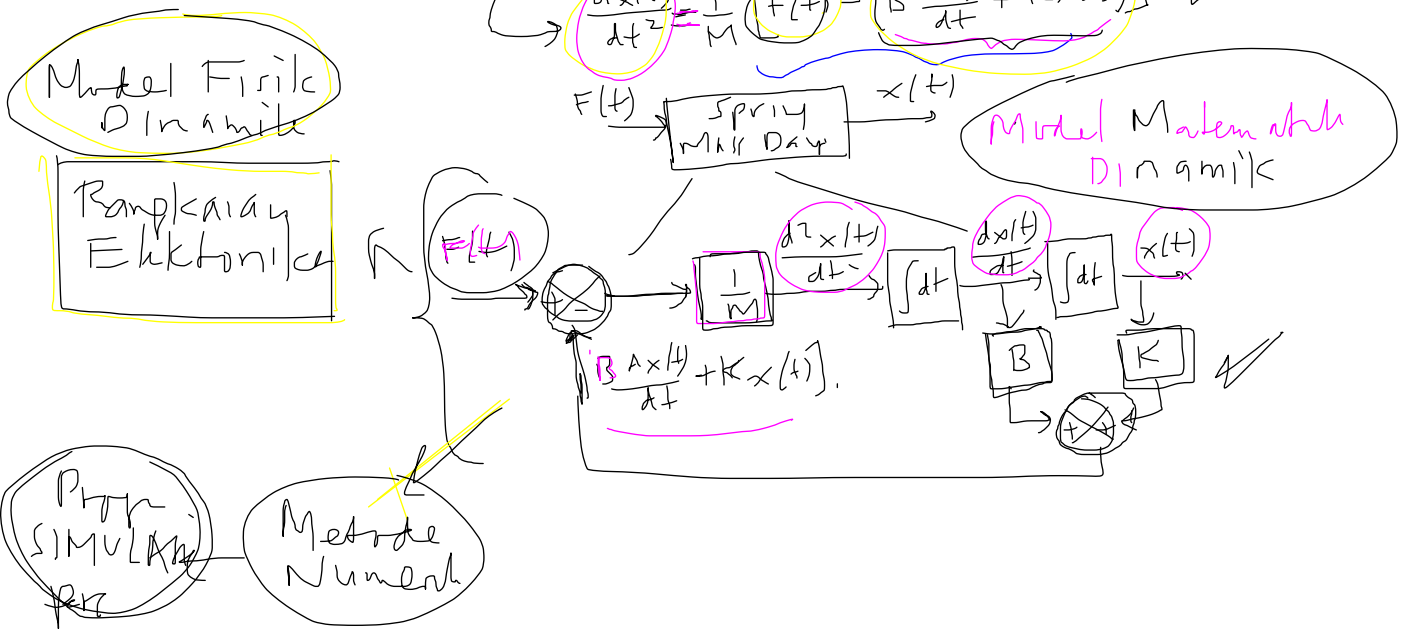
$$F(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t)$$

Pers Differensial

$$\frac{d^2x(t)}{dt^2} = \frac{1}{M}\left[F(t) - \left(B\frac{dx(t)}{dt} + Kx(t)\right)\right]$$

perceptan sesaat — kecepatan sesaat — posisi sesaat





$e = \frac{1}{0!}1^0 + \frac{1}{1!}1^1 + \frac{1}{2!}1^2 + \frac{1}{3!}1^3 + \ldots\ldots = 2,718\ldots$

Persamaan differensial $\frac{dx(t)}{dt} = -2x(t)$, $x(0) = 10$

Tentukan $x(t)$ secara analitik!

$x(t)$ adalah SOLUSI persamaan differensial di atas, dan karena dicari secara analitik maka hasilnya akan EXACT

$$\int \frac{dx(t)}{dt}\,dt = -2\int x(t)\,dt \quad \times$$

$$x(t) = -2\int x(t)\,dt$$

$$\int \frac{dx(t)}{x(t)} = -2\int dt$$

$$\ln[x(t)] = -2t + K$$

$$e^{\ln[x(t)]} = -2t + K$$

$$x(t) = e^{[-2t+K]}$$

$$= e^K e^{-2t} = A e^{-2t}$$ solusi umum

$$\int dx = x + K$$

$$\int x\,dx = \frac{1}{2}x^2 + K$$

$$\int x^2\,dx = \frac{1}{3}x^3 + K$$

$$\int x^{-2}\,dx = -x^{-1} + K$$

$$\int x^{-1}\,dx = \ln x + K$$

$$\int \frac{1}{x}\,dx = \ln x$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{dx(t)}{x(t)} = \ln x(t)$$

$$x(t) = A e^{-2t}$$

$$\frac{dx(t)}{dt} = -2A e^{-2t} = -2x(t)$$ checked

$t = 0$ $\quad x(0) = 10$

$$A e^{-2.(0)} = 10$$

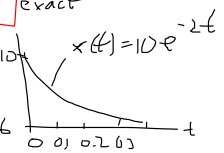
$$A = 10$$

Solusi analitik

$x(t) = 10e^{-2t}$ exact

t	x(t)
0	10
0,1	$10e^{-0,2}$
0,2	$10e^{-0,4}$
0,3	$10e^{-0,6}$

dst dst



Sebuah pers. diff. orde ke n.

$$a_n \frac{d^n x(t)}{dt^n} + a_{n-1}\frac{d^{n-1}x(t)}{dt^{n-1}} + a_{n-2}\frac{d^{n-2}x(t)}{dt^{n-2}} + \cdots + a_1 \frac{dx(t)}{dt} + a_0 x(t) = u(t)$$

x_2 $\quad x_1$

Dipecah menjadi n buah pers. diff. orde pertama :

$$x(t) = x_1 \longrightarrow \frac{dx(t)}{dt} = \frac{dx_1}{dt} = x_1 = x_2$$

$$\frac{dx(t)}{dt} = x_2 \longrightarrow \frac{d^2x(t)}{dt^2} = \frac{dx_2}{dt} = x_2 = x_3$$

$$\frac{d^{n-1}x(t)}{dt^{n-1}} = x_n \longrightarrow \frac{d^n x(t)}{dt^n} = -\frac{1}{a_n}\left[\quad\right] + \frac{1}{a_n}u(t)$$

$\dot{x}_n$ $\quad =$ $\quad$ (1)

$x_1 = x(t) \quad x_2 = \frac{dx(t)}{dt} \quad x_3 = \frac{d^2x(t)}{dt^2}$ ---

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \vdots \\ \frac{dx_n}{dt} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ & & & & & \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & -\frac{a_2}{a_n} & & \cdots & -\frac{a_{n-1}}{a_n} \end{bmatrix}}_{[n \times n]} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \frac{1}{a_n} \end{bmatrix}}_{[n \times 1]} u(t)$$

SISTEM n buah pers. diff pertama

Fungsi van der Pol.

$$\frac{d^2x(t)}{dt^2} + \mu\left[x(t)^2 - 1\right]\frac{dx(t)}{dt} + x(t) = 0$$

pers. differensial orde kedua

↓ 2 buah pers. diff orde pertama

$x(t) = x_1 \rightarrow \frac{dx(t)}{dt} = \frac{dx_1}{dt} = \dot{x}_1 = x_2$

$\frac{d^2x(t)}{dt^2} = \frac{dx_2}{dt} = \dot{x}_2$

$\dot{x}_2 + \mu\left[x_1^2 - 1\right]x_2 + x_1 = 0$

(1) $\frac{dx_1}{dt} = \dot{x}_1 = x_2$

(2) $\frac{dx_2}{dt} = \dot{x}_2 = -\mu\left[x_1^2 - 1\right]x_2 - x_1$

2 buah pers. diff orde pertama

Dapat diselesaikan dengan Metode Numerik orde ke 3