Numerical ——

Pemodelan & Metode Numerik

Pemodelan NUMERIK → RHz
Metode NUMERIK → Dr. Amil

website:

www.unhas.ac.id/rhiza/arsip/kuliah/

Penilaian:

$$NA = \frac{\text{Nilai RHz} + \text{Nilai AMIL}}{2}$$

(0 - 100) ↓

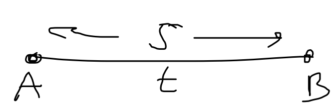
0 - 100 → A, A-, B+, B ..... dst
≥ 86 ——

→ Nilai RHz ↔ Tugas 1, 2, 3
Ujian FINAL

rhiza@unhas.ac.id
#6281643121 62

} 24 jam

* Time Continuum

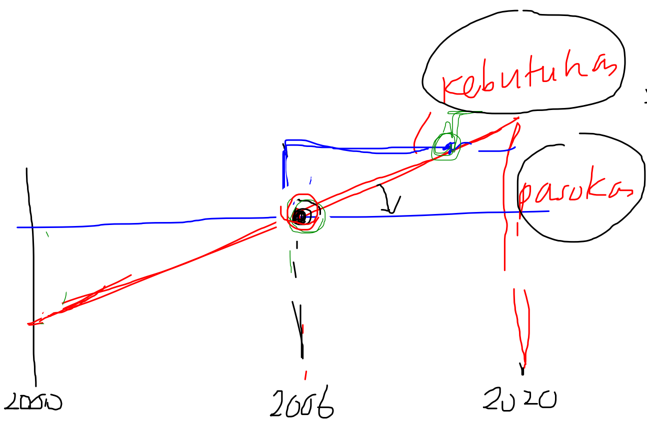


Kecepatan: $\frac{S}{t}$

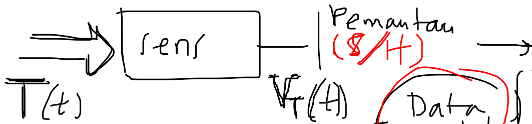
$$v(t) = \frac{dS}{dt}$$

Kontinyu

ANALOG



* Discrete-Time



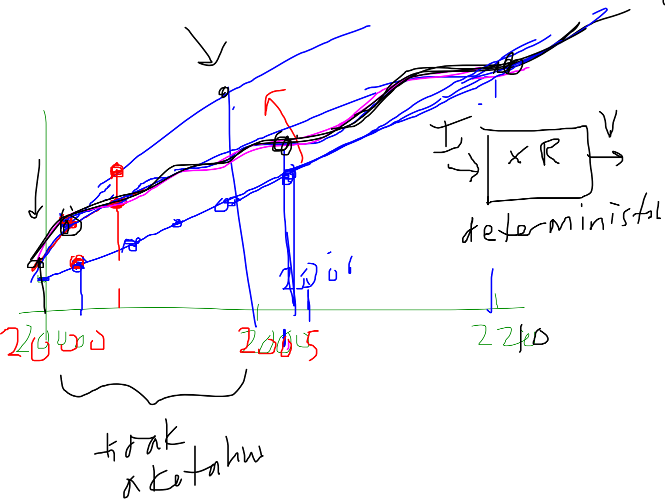
Waktu	Suhu
01.00	25
02.00	27
03.00	26

Discrete Time

Data Tercuplik

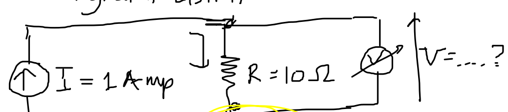
DIGITAL

T(t)'s ruangan



Contoh Sistem Deterministik & Stochastic

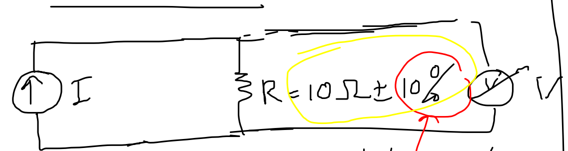
* Rangkaian Listrik



Hukum Ohm $V = I.R.$

I = 1 Amp → V = 1×10 = 10 Volt
I = 2 Amp → V = 2×10 = 20 Volt

(DETERMINISTIK)



toleransi

$9\Omega \leq R \leq 11\Omega$ → 10%

I = 1 Amp → 9 Volt ≤ V ≤ 11 Volt
I = 2 Amp → 18 Volt ≤ V ≤ 22 Volt

(STOKHASTIK)

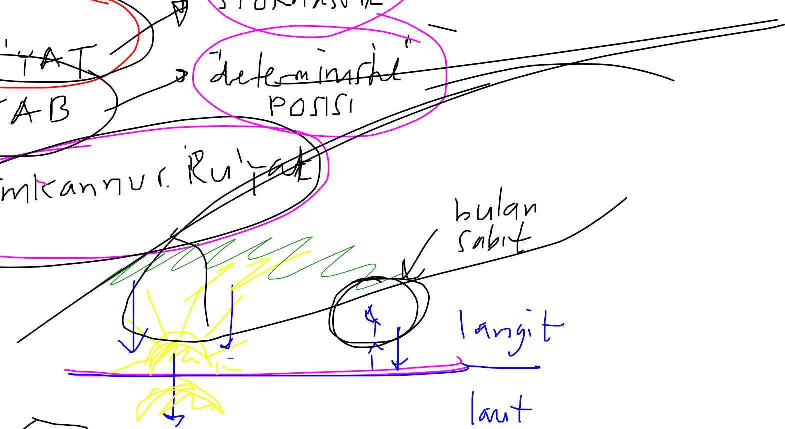
VISIBILITAS "stokhastik"

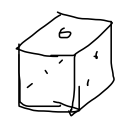
RU'YAT

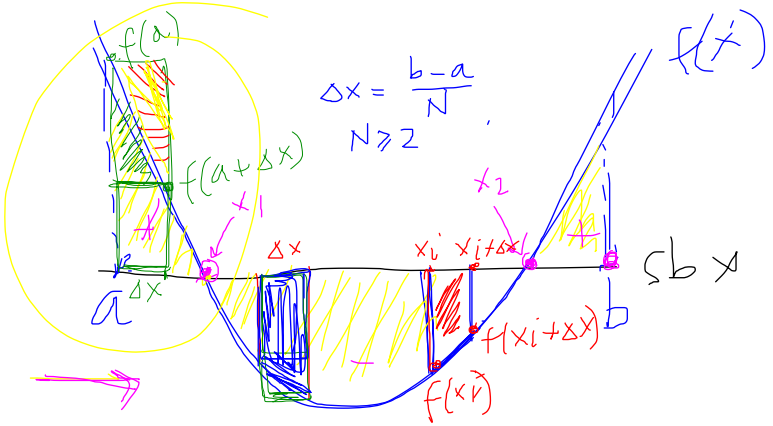
HISAB

"deterministik" POSISI

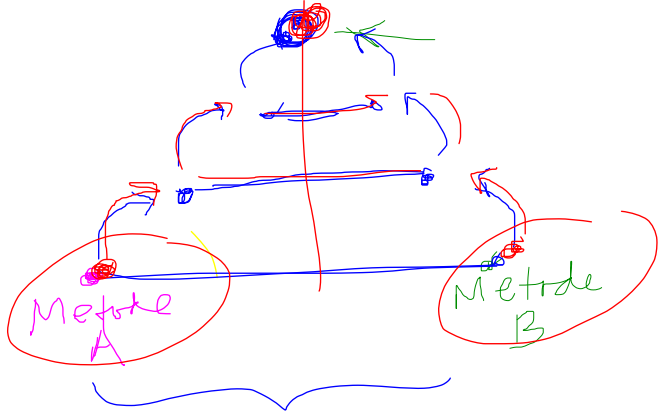
Imkannur Ru'yat





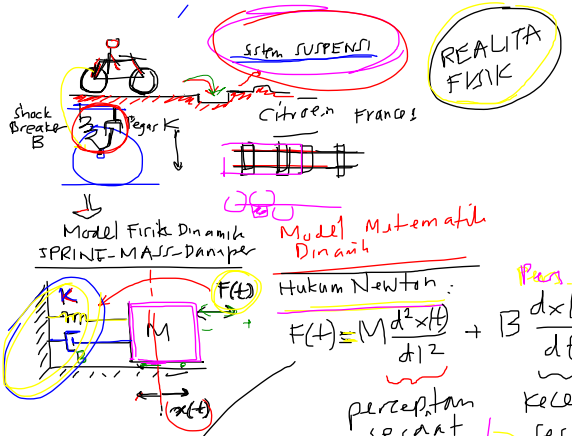


$$\Delta x = \frac{b-a}{N}, \quad N \geq 2$$



estimasi error

metode trapesium lebih bagus daripada metode 4-percep pangi



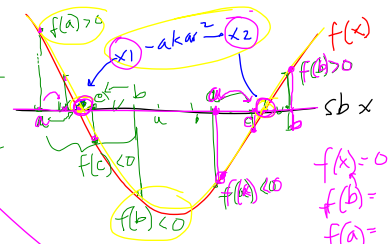
Hukum Newton :

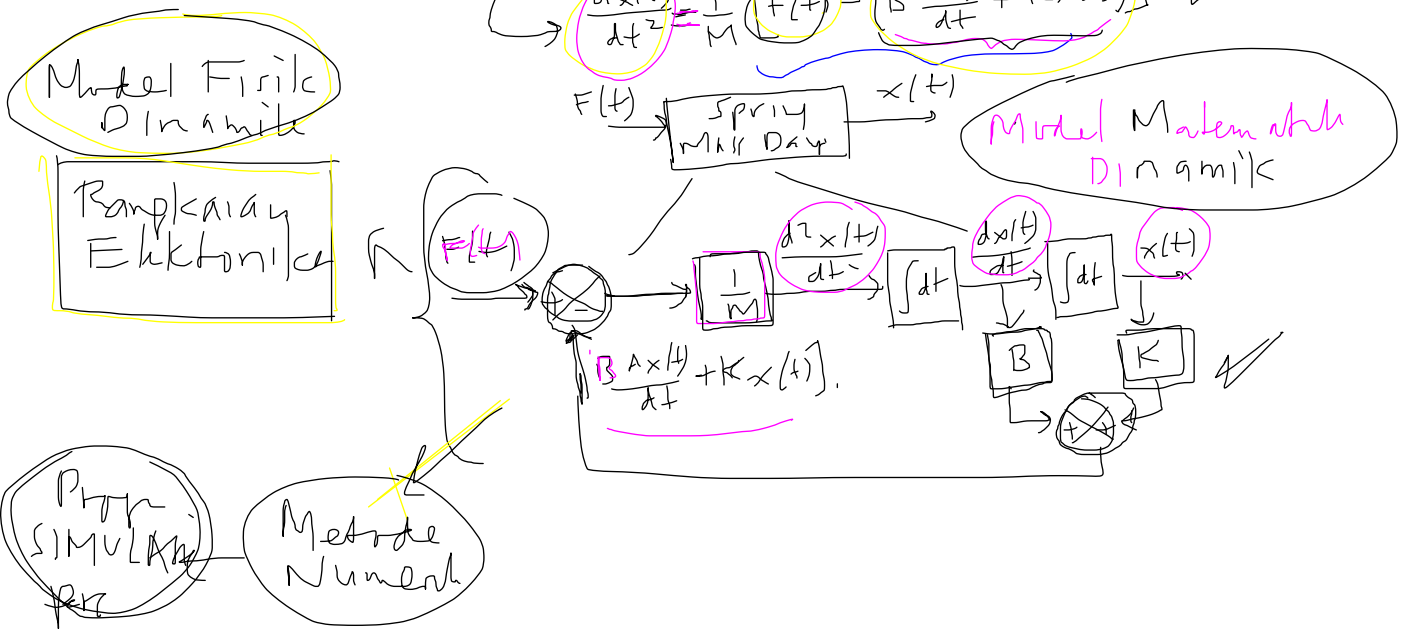
$$F(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t)$$

Pers Differensial

$$\frac{d^2x(t)}{dt^2} = \frac{1}{M}\left[F(t) - \left(B\frac{dx(t)}{dt} + Kx(t)\right)\right]$$

perceptan sesaat — kecepatan sesaat — posisi sesaat





$$e^1 = \frac{1}{0!}1^0 + \frac{1}{1!}1^1 + \frac{1}{2!}1^2 + \frac{1}{3!}1^3 + \dots = 2{,}718\dots$$

Persamaan differensial

$$\frac{dx(t)}{dt} = -2x(t), \quad x(0) = 10$$

Tentukan $x(t)$ secara analitik!

$x(t)$ adalah SOLUS persamaan differensial di atas, dan karena dicari secara analitik maka hasilnya akan EXACT

$$\int \frac{dx(t)}{dt}\,dt = -2\int x(t)\,dt$$

$$x(t) = -2\int x(t)\,dt \quad \times$$

$$\int \frac{dx(t)}{x(t)} = -2\int dt$$

$$\ln[x(t)] = -2t + K$$

$$e^{\log[x(t)]} = -t + K$$

$$x(t) = e^{[-2t+K]} = e^K e^{-2t} = A e^{-2t}$$

solusi umum

$$x(t) = A e^{-2t}$$

$$\frac{dx(t)}{dt} = -2Ae^{-2t} = -2x(t) \rightarrow \text{checked}$$

$t = 0$

$$x(0) = 10$$

$$Ae^{-2\cdot(0)} = 10$$

$$A = 10$$

$$\int \frac{dx}{x} =$$

$$\int dx = x + K$$

$$\int x\,dx = \frac{1}{2}x^2 + K$$

$$\int x^2\,dx = \frac{1}{3}x^3 + K$$

$$\int x^{-2}\,dx = -x^{-1} + K$$

$$\int x^{-1}\,dx = \ln x + K$$

$$\int \frac{1}{x}\,dx = \ln x$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{dx(t)}{x(t)} = \ln[x(t)]$$

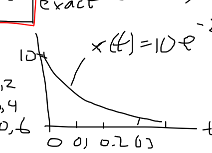
Solusi analitik

$$x(t) = 10e^{-2t} \quad \text{exact}$$

$t = 0, x(0) = 10$

$$\frac{dx(t)}{dt} = -2(10e^{-2t}) = -2x(t)$$

t	x(t)
0	10
0,1	$10e^{-0,2}$
0,2	$10e^{-0,4}$
0,3	$10e^{-0,6}$
dst	dst



Sebuah pers. diff. orde ke n.

$$a_n \frac{d^n x(t)}{dt^n} + a_{n-1}\frac{d^{n-1}x(t)}{dt^{n-1}} + a_{n-2}\frac{d^{n-2}x(t)}{dt^{n-2}} + \dots + a_1\frac{dx(t)}{dt} + a_0 x(t) = u(t)$$

Dipecah menjadi n buah pers. diff. orde pertama:

$$x(t) = x_1 \rightarrow \frac{dx(t)}{dt} = \frac{dx_1}{dt} = \dot{x}_1 = x_2$$

$$\frac{dx(t)}{dt} = x_2 \rightarrow \frac{d^2x(t)}{dt^2} = \frac{dx_2}{dt} = \dot{x}_2 = x_3$$

$$\vdots$$

$$\frac{d^{n-1}x(t)}{dt^{n-1}} = x_n \rightarrow \frac{d^n x(t)}{dt^n} = -\frac{1}{a_n}\left[\quad\right] + \frac{1}{a_n}u(t)$$

$$\dot{x}_n = \quad \text{—1—}$$

$x_1 = x(t) \quad x_2 = \frac{dx(t)}{dt} \quad x_3 = \frac{d^2x(t)}{dt^2}$ ---

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \vdots \\ \frac{dx_n}{dt} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ & & & & & \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & -\frac{a_2}{a_n} & & \cdots & -\frac{a_{n-1}}{a_n} \end{bmatrix}}_{[n \times n]} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \frac{1}{a_n} \end{bmatrix}}_{[n \times 1]} u(t)$$

SISTEM n buah pers. diff pertama

Fungsi van der Pol.

$$\frac{d^2x(t)}{dt^2} + \mu\left[x(t)^2 - 1\right]\frac{dx(t)}{dt} + x(t) = 0$$

pers. differensial orde kedua

↓ 2 buah pers. diff orde pertama

$x(t) = x_1 \rightarrow \frac{dx(t)}{dt} = \frac{dx_1}{dt} = \dot{x}_1 = x_2$

$\frac{d^2x(t)}{dt^2} = \frac{dx_2}{dt} = \dot{x}_2$

$\dot{x}_2 + \mu\left[x_1^2 - 1\right]x_2 + x_1 = 0$

(1) $\frac{dx_1}{dt} = \dot{x}_1 = x_2$

(2) $\frac{dx_2}{dt} = \dot{x}_2 = -\mu\left[x_1^2 - 1\right]x_2 - x_1$

2 buah pers. diff orde pertama

Dapat diselesaikan dengan Metode Numerik orde ke 3

Tugas 3

Pers Diff, $\frac{dx(t)}{dt} = -2x(t)$, $x(0)=10$

Solusi Analitik $x(t) = 10e^{-2t}$

t	x(t)	
0	10	
0,1	$10e^{-0,2}$ =	exact
0,2	$10e^{-0,4}$ =	
0,3	$10e^{-0,6}$ =	
⋮	⋮	
1	dst	

Program Numerik : set toleransi : misc. 1%

Algorithmia : * set $\Delta t = 0,1$

$t=0 \rightarrow x(0)=10$

solusi $x(0,1)$, 1st order = 8,00

$x(\Delta t)$ 2nd order = 8,20

estimasi error = $|8,20 - 8,00| = 0,2$

$= \frac{0,2}{10} \times 100\% = 2\% > 1\%$

$\Delta t = \frac{\Delta t}{n}$ → 2

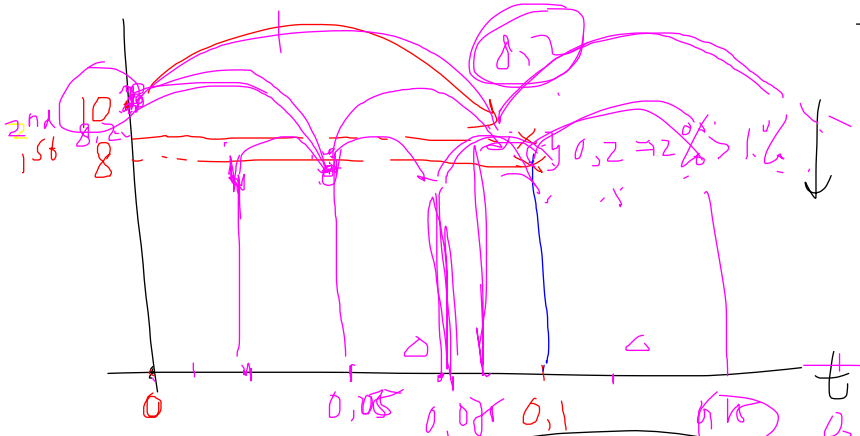
yes / no

solusi $x(\Delta t)$ didapat

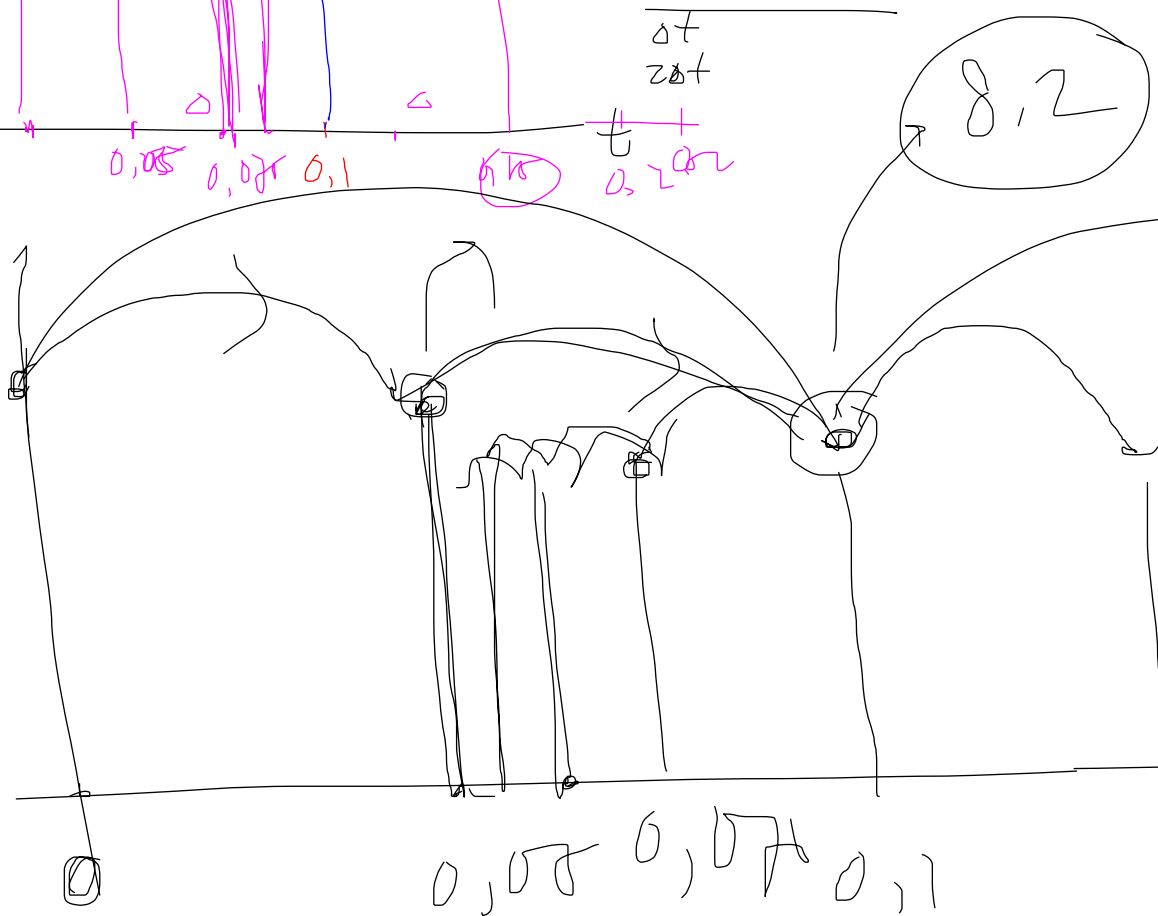
$t = \Delta t \rightarrow x(\Delta t) = \ldots$

$x(\Delta t + \Delta t)$: 1st order = …

2nd order = …



t	NUM x(t)
Δt	
2Δt	



t	x(t) NUM
0	10
0,1	8,2
0,15	—
0,175	—
0,2	
0,20125	

Pemodelan dan Metode Numerik

* Pemodelan Numerik pekan 1 - 8 RHZ
* Metode Numerik pekan 9 - 16 MBN

Referensi:

(1) Buku karangan Geoffrey Gordon
(2) Situs kuliah (course Website)

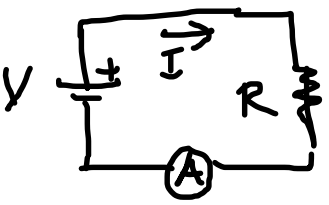
http://www.unhas.ac.id/rhiza/arsip/kuliah/

Penilaian

* Test → akhir semester
* Tugas 1, 2 dan (3) → .exe (cutable) file

→ N A RHZ
+
N A MBN

V = 10 Volt
R = 1 Ω



Deterministik

$$I = \frac{V}{R} = \frac{10}{1} = 10 \text{ Amp}$$

V = 10 Volt
R = 1 Ω ± 5 %
toleransi

Normal GAUSSIAN Distribution

$$I = \frac{V}{R} = \frac{10}{(1 \pm 0{,}05)}$$

$$\frac{10}{1{,}05} \leq I \leq \frac{10}{0{,}95}$$

stokhastik