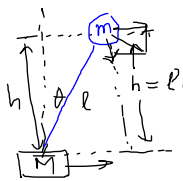
$$T_2 = \frac{1}{2}m\left[x_2 + \ell x_4 \cos x_3\right]^2 + \frac{1}{2}m\left[\ell x_4 \sin x_3\right]^2$$
$$= \frac{1}{2}m\left(x_2^2 + \ell^2 x_4^2 \cos^2 x_3 + \ell^2 x_4^2 \sin^2 x_3 + 2x_2 \ell x_4 \cos x_3\right)$$
$$= \frac{1}{2}m x_2^2 + \frac{1}{2}m\ell^2 x_4^2 + m x_2 \ell x_4 \cos x_3$$



$V = mgh = mg\ell\cos\theta$

$T = T_1 + T_2$

$T_1 = \frac{1}{2}M x_2^2$

$T_2 = \frac{1}{2}m\left(V_x^2 + V_y^2\right)$

$V_x = \frac{d}{dt}\left[x(t) + \ell\sin\theta(t)\right]$

posisi serasah arah x dari m

$= \frac{dx(t)}{dt} + \ell\cos\theta(t)\cdot\frac{d\theta(t)}{dt}$

$= x_2 + \left[\ell(\cos x_3)\right]\cdot x_4$

$V_y = \frac{d}{dt}\left[\ell\cos\theta(t)\right]$ (posisi serasah arah y)

$V_y = \left[\ell\sin\theta(t)\right]\cdot\frac{d\theta(t)}{dt}$

$= \left[\ell\sin x_3\right]x_4$

$T = T_1 + T_2$

$= \frac{1}{2}M x_2^2 + \frac{1}{2}m x_2^2 + \frac{1}{2}m\ell^2 x_4^2 + m x_2 x_4 \ell\cos x_3$

$V = mg\ell\cos x_3$

* Penurunan fungsi $f_1(t)$ dan $f_2(t)$

Persamaan Lagrange : $L = T - V$

T = Energi Kinetik

V = Energi Potensial

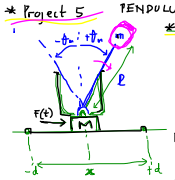
Arah x (linier) $\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{x}_1}\right] - \frac{\partial L}{\partial x_1} = u = F(t)$

Arah θ (melingkar) : $\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{x}_3}\right] - \frac{\partial L}{\partial x_3} = 0$

$\dot{x}_1 = x_2 = \frac{dx(t)}{dt}$

$\dot{x}_3 = x_4 = \frac{d\theta(t)}{dt}$

* Project 5 PENDULUM TERBALIK



* Desain konstanta

d = ... m

M = ... kg

m = ... kg

M > m

ℓ = ... m

±θm = ...

Posisi awal

x = 0

θ = +θm atau -θm

Gaya F(t) +tanan

- Kiri

* State assignment

$u = F(t)$

$x_1 = x(t)$, posisi M

$x_2 = \dot{x}_1 = \frac{dx(t)}{dt}$, kecepatan M

$x_3 = \theta(t)$ & posisi sudut ℓ

$x_4 = \dot{x}_3 = \frac{d\theta(t)}{dt}$, kecepatan sudut ℓ

* Persamaan keadaan.

$\dot{x}_1 = x_2$

$\dot{x}_2 =$ fungsi $f_1(t) = \frac{d^2x(t)}{dt^2}$

$\dot{x}_3 = x_4$

$\dot{x}_4 =$ fungsi $f_2(t) = \frac{d^2\theta(t)}{dt^2}$

(tak linier, tak stabil)

$L = T - V = \frac{1}{2}(M+m)\dot{x}_2^2 + \frac{1}{2}m\ell^2 x_4^2 + m x_2 x_4 \ell\cos x_3 - mg\ell\cos x_3$

arah linier $\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{x}_2}\right] - \frac{\partial L}{\partial x_1} = u = F(t)$

$\frac{d}{dt}\left[(M+m)\dot{x}_2 + m\ell x_4\cos x_3\right] = F(t)$

$(M+m)\ddot{x}_2 + m\ell\left[\dot{x}_4\cos x_3 - x_4(\sin x_3)\dot{x}_3\right] = F(t)$ ($\dot{x}_3 = x_4$)

$(M+m)\ddot{x}_2 + m\ell\dot{x}_4\cos x_3 - m\ell x_4^2\sin x_3 = F(t)$ —— (1)

arah melingkar : $\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{x}_4}\right] - \frac{\partial L}{\partial x_3} = 0$

$\frac{d}{dt}\left[m\ell^2 x_4 + m\ell(\cos x_3)x_2\right] - \left(-m x_2 x_4 \ell\sin x_3\right) + mg\ell(\sin x_3) = 0$

$m\ell^2\dot{x}_4 + m\ell\dot{x}_2\cos x_3 - m\ell x_2(\sin x_3)\dot{x}_3 + m\ell x_2 x_4\sin x_3 - mg\ell\sin x_3 = 0$ ($\dot{x}_3 = x_4$)

$m\ell^2\dot{x}_4 + m\ell\dot{x}_2\cos x_3 - mg\ell\sin x_3 = 0$ —— (2)

Dari pers (1) dan (2) diperoleh :

a $\ddot{x}_2 = \frac{F(t) - \frac{1}{2}mg\sin 2x_3 + m\ell x_4^2\sin x_3}{M + m\sin^2 x_3}$

b $\dot{x}_4 = \frac{g\sin x_3 - \ddot{x}_2\cos x_3}{\ell}$

} Matlab Function

Selanjutnya : Desain, menentukan gain matrix

$K = [K_1\ K_2\ K_3\ K_4]$