

$F(s) = \frac{1}{s^2} \rightarrow f(t) = t$

 $F(z) = Tz^{-1} + 2Tz^{-2} + 3Tz^{-3} + \dots$

Contoh (tanpa Laplace)
Diketahui sebuah Isyarat
duplikat

k	$x(k)$
0	2
1	0
2	2
3	0
4	2
>4	0

Tentukan $X(z) = \sum x(k)$!

$$X(z) = \sum_{k=0}^{\infty} z^{-k} x(k) = z^{-0} x(0) + z^{-1} x(1) + z^{-2} x(2) + z^{-3} x(3) + z^{-4} x(4) + \dots$$

$$X(z) = \frac{z(z^4 + z^2 + 1)}{z^4}$$

* Transformasi z
(Ragazzini & Zadeh, 1952)

Definisi $Zf(k) = \sum_{k=0}^{\infty} z^{-k} f(k)$ $k=0,1,2,3,-$

Contoh

Isyarat makan ratuwan yang disampling dengan periode T

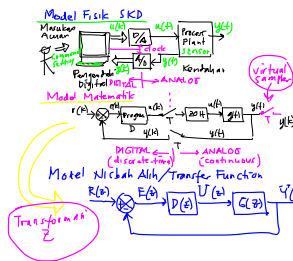
$u(k) = \begin{cases} 0, & k < 0 \\ 1, & k \geq 0 \end{cases}$

$k: 0, 1, 2, 3, \dots$

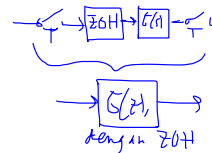
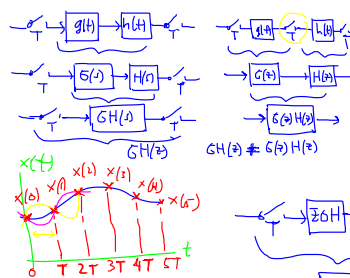
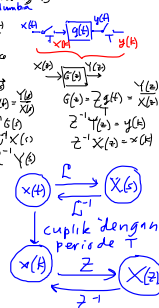


$$\begin{aligned} U(z) &= \sum_{k=0}^{\infty} z^{-k} u(k) = z^{-0} u(0) + z^{-1} u(1) + z^{-2} u(2) + \dots \\ &= 1 + z^{-1} + z^{-2} + z^{-3} + \dots \\ &= 1 + z^{-1} [1 + z^{-1} + z^{-2} + z^{-3} + \dots] \\ &= 1 + z^{-1} U(z) \end{aligned}$$

$$(1 - z^{-1})U(z) = 1 \rightarrow U(z) = \frac{1}{1 - z^{-1}} = \frac{z}{z - 1}$$



* TRANSFORMASI Z
(Prof. Ragozzini) 195
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Contoh: $G(r) = \frac{1}{r+1}$

* $U(s) \xrightarrow{\mathcal{L}^{-1}} Y(s) \xrightarrow{\mathcal{L}^{-1}} y(kT)$
 $U(s) = \frac{1}{s} \rightarrow Y(s) = \frac{1}{s(s+2)}$
 $\mathcal{L}^{-1} \left\{ \frac{1}{s(s+2)} \right\} = \frac{1}{2} (1 - e^{-2t})$

Misalnya $T = 1$ detik

k	t	y(kT)
0	0	0
1	1	$\frac{1}{2}(1 - e^{-2}) = 0,4323$
2	2	$\frac{1}{2}(1 - e^{-4}) = 0,4908$
3	3	$\frac{1}{2}(1 - e^{-6}) = 0,4988$
4	4	$\frac{1}{2}(1 - e^{-8}) = 0,4998$
5	5	$\frac{1}{2}(1 - e^{-10}) = 0,4999$

* Transformasi

[illegible]

$y(k) = Z^{-1}Y(z)$ dapat diperoleh dari pembagian:

$$\frac{1 + 1,1353z + 0,1353z^2}{z^2 - 1,1353z + 0,1353}$$

$$\frac{1,1353z + 0,1353}{z^2 - 1,1353z + 0,1353}$$

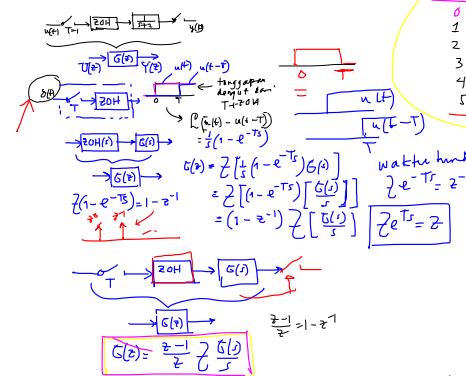
$$\frac{1,1353z - 0,1353z^2 + 0,1561z^2}{z^2 - 1,1353z + 0,1353}$$

$$\frac{1,1561z^2}{z^2 - 1,1353z + 0,1353}$$

$$\frac{1,1561z^2 - 1,3097z + 0,7561z^2}{z^2 - 1,1353z + 0,1353}$$

$$\frac{1,1561z^2 - 1,3097z + 0,7561z^2 + 1,1561z - 1,1561z^2 + 0,1561z^2}{z^2 - 1,1353z + 0,1353}$$

$$\frac{1,1561z^2 - 1,3097z + 0,7561z^2 + 1,1561z - 1,1561z^2 + 0,1561z^2}{z^2 - 1,1353z + 0,1353}$$



$$G(z) = \frac{1}{s+2}, \quad s \neq -1 \rightarrow G(z) = \frac{z-1}{z} \cdot \frac{1}{s(s+2)}$$

$$= \frac{1}{z} \left[\frac{0,8647z}{(z-1)(z-0,1353)} \right]$$

$$= \frac{1}{z} \left[\frac{0,8647}{z-0,1353} \right] = \frac{0,4323}{z-0,1353}$$

$$Y(z) = \tilde{C}(z)U(z) = \left(\frac{0,432z}{z-0,135} \right) \left(\frac{z}{z-1} \right)$$

k	$y(k)$
0	0
1	0,4323
2	0,4908
3	0,4902

Jadi, $G(z)$ yang diperoleh dengan menambahkan ZOH lebih sesuai dengan "realita" daripada yang tanpa ZOH