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Nomor (2)

c) $A = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$

nilai eigen

$\hookrightarrow \det [\lambda I - A] = 0$

$\det \left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \right) = 0$

$\det \begin{bmatrix} \lambda-1 & 0 \\ -1 & \lambda+1 \end{bmatrix} = 0$

$(\lambda-1)(\lambda+1) = 0$

$\lambda_1 = 1$

$\lambda_2 = -1$

~~↳ karena salah satu nilai eigennya bagian real bernilai negatif maka sistem dikatakan stabil~~

d) Transformasi Similaritas

$A = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$

$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$T = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 1 \end{bmatrix}$

$D = [1]$

maka,

$\hat{A} = TAT^{-1} = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$

$\hat{A} = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 0.5 \end{bmatrix}$

$\hat{A} = \begin{bmatrix} 1 & 0 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 0.5 \end{bmatrix}$

$\hat{A} = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$

$\hat{B} = TB = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

$\hat{C} = CT^{-1} = [1 \ 1] \begin{bmatrix} 1 & 0 \\ -1 & 0.5 \end{bmatrix} = [0 \ 0.5]$

$\hat{D} = D = [1]$

e) nilai eigen dari $\hat{A}$ = nilai eigen matriks TAT^{-1}

$\hookrightarrow \det [\lambda I - \hat{A}] = 0$

$\det \left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix} \right) = 0$

$\det \begin{bmatrix} \lambda-1 & 0 \\ -6 & \lambda+1 \end{bmatrix} = 0$

$(\lambda-1)(\lambda+1) = 0$

$\lambda_{1,2} = \pm 1$

$\hookrightarrow \det [\hat{\lambda} I - \hat{A}] = 0$

$\det \left(\begin{bmatrix} \hat{\lambda} & 0 \\ 0 & \hat{\lambda} \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix} \right) = 0$

$\det \begin{bmatrix} \hat{\lambda}-1 & 0 \\ -6 & \hat{\lambda}+1 \end{bmatrix} = 0$

$(\hat{\lambda}-1)(\hat{\lambda}+1) = 0$

$\hat{\lambda}_{1,2} = \pm 1$

⇨ sama ⇨

f) $\hat{\dot{x}} = \hat{A}\hat{x} + \hat{B}u$
 $y = \hat{C}\hat{x} + \hat{D}u$

$\hat{\dot{x}} = \begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix} \hat{x} + \begin{bmatrix} 1 \\ 4 \end{bmatrix} u$
 $y = [0 \ 0.5] \hat{x} + [1] u$

$\dot{\hat{x}}_1 = \hat{x}_1 + u$
 $\dot{\hat{x}}_2 = 6\hat{x}_1 - \hat{x}_2 + 4u$
 $y = 0.5\hat{x}_2 + u$