DASAR SISTEM KENDALI → SISTEM LINIER → SISTEM KENDALI → Peranc SISTEM KENDALI → SKRIPSI

3 (umum) 4 (TK) 5 (TK) 7

337D4103 SISTEM KENDALI

Ralat Jadwal

3SKS. * 1 SKS : pekan 1 s/d 8 RHZ
1 SKS . pekan 9 s/d 16 AEU
1 SKS . PRAKTIKUM.
- Praktikum Individu
- Praktikum Kelompok

3SKS

Penilaian N.A = $\dfrac{\text{RHZ} + \text{AEU} + \text{Praktikum}}{3}$ ← kurang lebih

Link : http://www.unhas.ac.id/rhiza/arsip/kuliah/

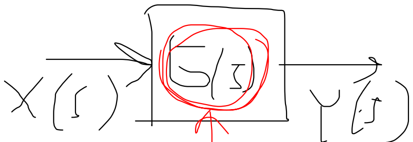
RHZ : TEST 1 (pekan ke-8)
TEST 2 (pekan ke-16)

→ Nilai RHZ = $\dfrac{\text{Test I} + \text{Test 2}}{2}$

Silabus dapat dilihat di :

http://www.unhas.ac.id/rhiza/arsip/kuliah/silabus-2008/

* Hanya memperlihatkan hubungan input – output saja, tidak memperlihatkan DINAMIKA INTERNAL sistem



$$G(s) = \frac{Y(s)}{X(s)}$$

* Model Ruang Keadaan bersifat lebih UMUM (general) ~~dari Model~~ Nisbah Alih: semua sistem yang dapat dimodelkan dengan model Nisbah Alih akan dapat dimodelkan dengan Model Ruang Keadaan, tapi belum tentu sebaliknya.

* Model Nisbah Alih hanya dapat memodelkan sistem linier. Model Ruang Keadaan dapat memodelkan sistem Tak Linier juga.

LTV

LTI saja

Apa kelebihan Model Nisbah Alih??

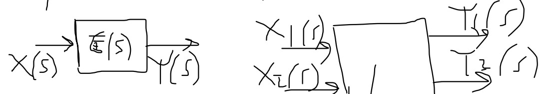
Praktis dan sederhana

Model Ruang Keadaan

State Space Model

Kelemahan Model Nisbah Alih (Transfer Function):

* Hanya cocok untuk sistem "sederhana", SISO



4 (empat) Nisbah Alih

$$G_1(s) = \frac{Y_1(s)}{X_1(s)}\bigg|_{X_2(s)=0}$$

$$G_2(s) = \frac{Y_2(s)}{X_1(s)}\bigg|_{X_2(s)=0}$$

$$G_3(s) = \frac{Y_1(s)}{X_2(s)}\bigg|_{X_1(s)=0}$$

$$G_4(s) = \frac{Y_2(s)}{X_2(s)}\bigg|_{X_1(s)=0}$$

Model Ruang Keadaan bisa untuk sistem MIMO

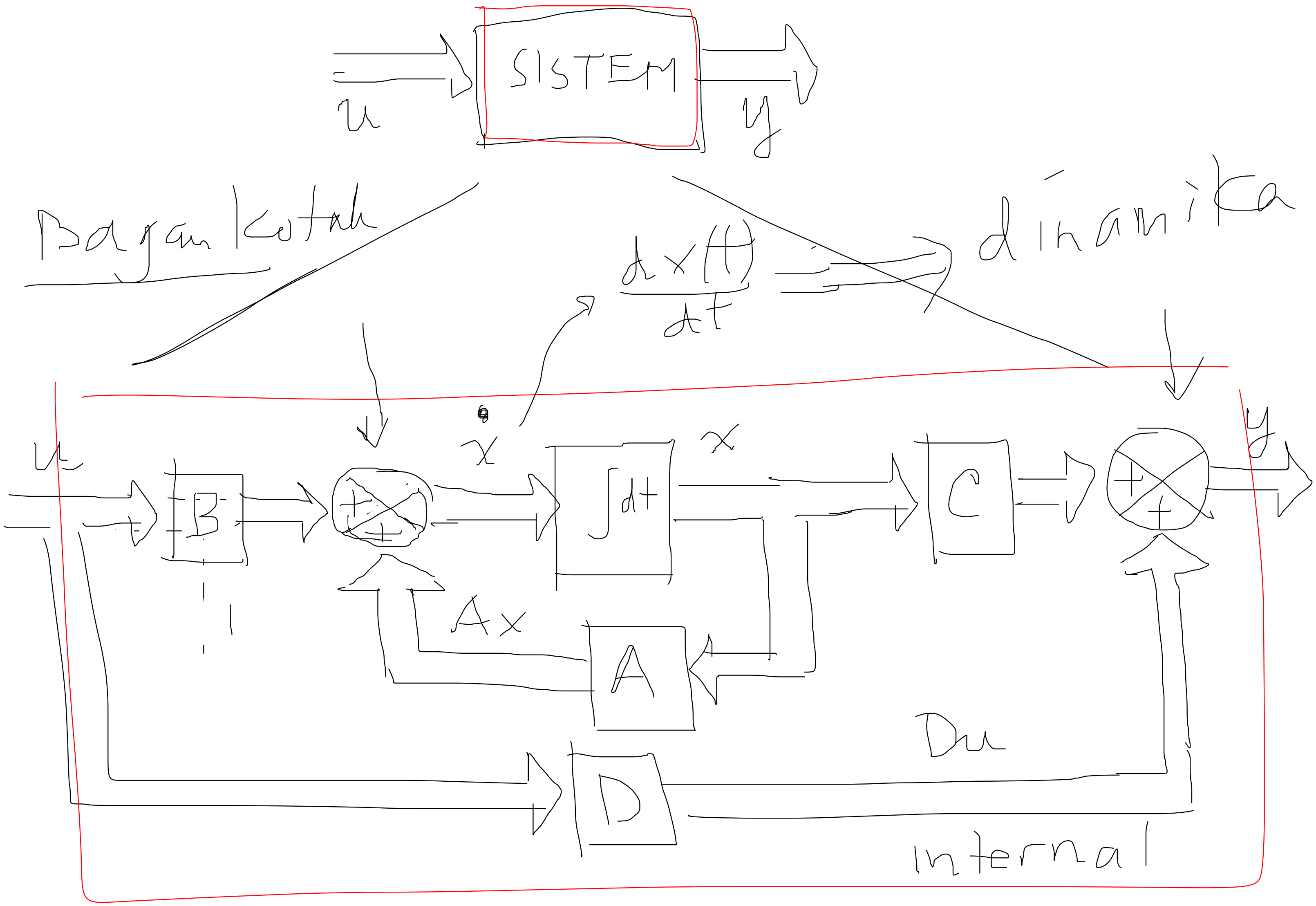
* Analisis dengan model Nisbah Alih lebih efektif untuk "pencil & paper", sulit untuk simulasi dengan komputer

Tabel Laplace

$$\mathcal{L}^{-1}\frac{1}{s+2} = \ldots ? \rightarrow e^{-2t}$$

$$\frac{1}{s+a} \rightarrow e^{-at}$$

Model Ruang Keadaan (State space)



Persamaan: Keadaan

(1) Persamaan (Masukan):

(2) Persamaan Keluaran:

$$\boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}}$$

Aljabar Matrix (Aljabar Linier)

$$\underset{[n\times 1]}{\dot{x}} = \underset{[n\times n]}{A}\underset{[n\times 1]}{x} + \underset{[n\times m]}{B}\underset{[m\times 1]}{u}$$

Dimensi: $A[n\times n]$, $B[n\times m]$

$$\underset{[k\times 1]}{y} = \underset{[k\times n]}{C}\underset{[n\times 1]}{x} + \underset{[k\times 1]}{Du}$$

$C[k\times n]$, $D[k\times m]$

Definisi:

$u \triangleq u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}$ vektor kolom $[m\times 1]$

masukan INPUT

$y \triangleq y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_k(t) \end{bmatrix}$ vektor kolom $[k\times 1]$

keluaran OUTPUT

$x \triangleq x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$ vektor kolom $[n\times 1]$

peubah keadaan (state variable)

$\dot{x} \triangleq \dfrac{dx(t)}{dt} = \begin{bmatrix} dx_1(t)/dt \\ dx_2(t)/dt \\ \vdots \\ dx_n(t)/dt \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix}$ vektor kolom $(n\times 1)$

Contoh: Suatu sistem mempunyai 3 masukan, 4 keluaran dan 5 peubah keadaan. Tentukan dimensi matrix A, B, C dan D

Jawab: $m = 3$, $n = 5$, $k = 4$

$A[n\times n] \longrightarrow A[5\times 5]$

$B[n\times m] \longrightarrow B[5\times 3]$

$C[k\times n] \longrightarrow C[4\times 5]$

$D[k\times m] \longrightarrow D[4\times 3]$

Catatan:

* Jika matrix A, B, C dan D semuanya berisi KONSTANTA, maka sistem linier yang "time-invariant" (LTI)

* Jika di antara matrix A, B, C dan D ada yang berisi peubah fungsi waktu t, maka sistem linier "time-varying" (LTV)

* Selain itu sistem tak linier

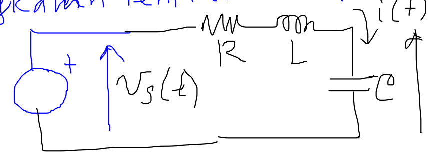
* Model Nisbah Alih hanya dapat memodelkan sistem LTI saja

Next Contoh

Next
ss2tf !
tf2ss 0

* Contoh Elektrik

Rangkaian Pemuatan Kapasitor (RLC seri)



Susunlah Model Ruang Keadaan.

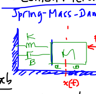
$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

dengan penunjukan (state assignment)

$$u \triangleq v_s(t)$$
$$y_1 \triangleq v_C(t)$$
$$y_2 \triangleq i(t)$$
$$x_1 \triangleq i(t)$$
$$x_2 \triangleq v_C(t)$$

* Contoh Mekanik.

Spring-Mass-Damper System



Susunlah Model Ruang Keadaan
dengan: $u \triangleq F(t)$, 1 masukan, $m = 1$
$y \triangleq x(t)$, 1 keluaran $k = 1$

$\begin{bmatrix} \dot{x} = Ax + Bu \\ y = Cx + Du \end{bmatrix}$ $\left.\begin{matrix} x_1 \triangleq \frac{dx(t)}{dt} \\ x_2 \triangleq x(t) \end{matrix}\right\}$, 2 peubah keadaan $n = 2$

Jawab

* Dimensi matrix $A[2\times 2]$, $B[2\times 1]$, $C[1\times 2]$, $D[1\times 1]$

* Hukum Newton:

$$F(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t)$$

$$x(t) = x_2 \rightarrow \frac{dx(t)}{dt} = \dot{x}_2 \rightarrow \dot{x}_2 = x_1$$

$$\frac{dx(t)}{dt} = x_1 \rightarrow \frac{d^2x(t)}{dt^2} = \dot{x}_1$$

$$u = M\dot{x}_1 + Bx_1 + Kx_2$$

$$\dot{x}_1 = -\frac{B}{M}x_1 - \frac{K}{M}x_2 + \frac{1}{M}u$$

$$y = x(t) \longrightarrow y = x_2$$

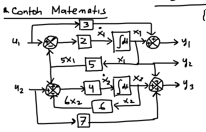
$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{B}{M} & -\frac{K}{M} \\ 1 & \emptyset \end{bmatrix}}_{A[2\times 2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{M} \\ \emptyset \end{bmatrix}}_{B[2\times 1]} u$$

$$y = \underbrace{\begin{bmatrix} \emptyset & 1 \end{bmatrix}}_{C[1\times 2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{[\emptyset]}_{D[1\times 1]} u$$

Jadi: $A = \begin{bmatrix} -\frac{B}{M} & -\frac{K}{M} \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1/M \\ 0 \end{bmatrix}$

$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$ $D = [0]$

* Contoh Matematis



Susunlah:
Model Ruang Keadaan

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

a) Dimensi Matrix A, B, C dan D

* jumlah masukan $m = 2$
* jumlah keluaran $k = 3$
* jumlah peubah keadaan $n = 2$

$A[2\times 2]$, $B[2\times 2]$, $C[3\times 2]$, $D[3\times 2]$

Model Ruang Keadaan $\begin{bmatrix} \dot{x} = Ax + Bu \\ y = Cx + Du \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

b) Pers Keadaan

$$\dot{x}_1 = 2[-5x_1 + u_1] = -10x_1 + 2u_1$$
$$\dot{x}_2 = 4[+5x_1 - 6x_2 + u_2]$$
$$= 20x_1 - 24x_2 + 4u_2$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -10 & 0 \\ 20 & -24 \end{bmatrix}}_{A[2\times 2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}}_{B[2\times 2]} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$A = \begin{bmatrix} -10 & 0 \\ 20 & -24 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$

c) Pers. keluaran

$$\left.\begin{matrix} y_1 = x_1 + 3u_1 \\ y_2 = x_1 \\ y_3 = x_1 + x_2 + 7u_2 \end{matrix}\right\} y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}}_{C[3\times 2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 3 & 0 \\ 0 & 0 \\ 0 & 7 \end{bmatrix}}_{D[3\times 2]} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$C = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \\ 0 & 7 \end{bmatrix}$

Hukum Ohm.

(1) $v_s(t) - v_C(t) = L\frac{di(t)}{dt} + Ri(t)$

(2) $i(t) = C\frac{dv_C(t)}{dt}$

* Dimensi Matrix.

$m = 1$ $k = 2$ $n = 2$

$A[2\times 2]$, $B = [2\times 1]$, $C[2\times 2]$, $D[2\times 1]$

* Persamaan Keadaan

$x_1 \triangleq i(t) \rightarrow \frac{di(t)}{dt} = \dot{x}_1$

$x_2 \triangleq v_C(t) \rightarrow \frac{dv_C(t)}{dt} = \dot{x}_2$

(1) $u - x_2 = L\dot{x}_1 + Rx_1$

$$\dot{x}_1 = -\frac{R}{L}x_1 - \frac{1}{L}x_2 + \frac{1}{L}u$$

(2) $x_1 = C\dot{x}_2 \longrightarrow \dot{x}_2 = \frac{1}{C}x_1$

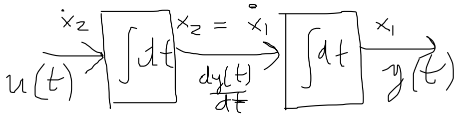
$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & \emptyset \end{bmatrix}}_{A[2\times 2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{L} \\ \emptyset \end{bmatrix}}_{B[2\times 1]} u$$

* Persamaan Keluaran

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \emptyset & 1 \\ 1 & \emptyset \end{bmatrix}}_{C[2\times 2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix}}_{D[2\times 1]} u$$

$y_1 = v_C(t) \rightarrow y_1 = x_2$
$y_2 = i(t) \rightarrow y_2 = x_1$

Contoh: "Double Integrator"



State assignment:

$x_1 \triangleq y(t) \rightarrow \dot{x}_1 = \frac{dy(t)}{dt}$

$x_2 \triangleq \frac{dy(t)}{dt} \quad \dot{x}_1 = x_2$

$u \triangleq \dot{x}_2 \quad \dot{x}_2 = u$

$y = y(t) = x_1$

Pers. keadaan

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A[2\times 2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B[2\times 1]} u$$

Pers. keluaran:

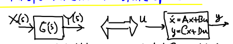
$$y = \underbrace{\begin{bmatrix} 1 & \emptyset \end{bmatrix}}_{C[1\times 2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{[\emptyset]}_{D[1\times 1]} u$$

ss2tf(A,B,C,D)

Model Ruang Keadaan dari "Double Integrator"

>>> help ss2tf
>>> help tf2ss
MATLAB functions Control Tool Box

"State Space" to "Transfer Function"
"Transfer Function" to "State Space"



Model Nisbah Alih (Transfer Function) ⟺ Model Ruang Keadaan (State Space)

Semua Model Nisbah Alih dapat dibuat Model Ruang Keadaan, tapi tidak sebaliknya.

* ss2tf Model Ruang Keadaan yang dapat diubah menjadi model Nisbah Alih hanya model sistem SISO (single input single output)

⟹ $A[n\times n]$, $B[n\times 1]$, $C[1\times n]$, $D[1\times 1]$
$m = 1$ dan $k = 1$ (SISO)

$\dot{x} = Ax + Bu \qquad u \triangleq u(t)$ single input

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \vdots \\ \frac{dx_n(t)}{dt} \end{bmatrix} = \begin{bmatrix} & & \\ & A[n\times n] & \\ & & \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{B[n\times 1]} u(t)$$

Transf. Laplace:

$$\begin{bmatrix} sX_1(s) \\ sX_2(s) \\ \vdots \\ sX_n(s) \end{bmatrix} = A \begin{bmatrix} X_1(s) \\ X_2(s) \\ \vdots \\ X_3(s) \end{bmatrix} + BU(s)$$

$$\underbrace{\begin{bmatrix} sX_1(s) \\ sX_2(s) \\ \vdots \\ sX_n(s) \end{bmatrix}}_{[n\times 1]} - \underset{[n\times n]}{A} \underbrace{\begin{bmatrix} X_1(s) \\ X_2(s) \\ \vdots \\ X_n(s) \end{bmatrix}}_{[n\times 1]} = \underset{[n\times n]}{B}\,\underset{[1\times 1]}{U(s)}$$

$$sIX(s) - AX(s) = BU(s)$$

$$I = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & \ddots \\ & & & 1 \end{bmatrix}$$

$$sI = \underbrace{\begin{bmatrix} s & & & 0 \\ & s & & \\ & & s & \\ 0 & & & \ddots \\ & & & & s \end{bmatrix}}_{[n\times n]}$$

$sIX(s) - AX(s) = BU(s)$

$\underbrace{[sI - A]X(s)}_{} = BU(s)$

$\underbrace{[sI-A]^{-1}[sI-A]}_{I}X(s) = [sI-A]^{-1}BU(s)$

$X(s) = [sI-A]^{-1}BU(s)$

$y = Cx + Du$

Transf. Laplace:

$$Y(s) = CX(s) + DU(s)$$

$$= C[sI-A]^{-1}BU(s) + DU(s)$$

$$= \left[C[sI-A]^{-1}B + D\right]U(s)$$

Transfer Function:

$$\boxed{G(s) = \left[C[sI-A]^{-1}B + D\right]}$$

Khusus SISO

Model Nisbah Alih:

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\operatorname{adj}[sI-A]}{|I - A|} = \frac{\begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix}}{s^2} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$G(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0$$

$$= \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0 = \frac{1}{s^2}$$

Sebagai contoh, berikut ini adalah penentuan peubah keadaan yang menghasilkan bentuk matrix A khusus yang disebut "Jordan Companion matrix"

$u \triangleq u(t)$, masukan $\quad U(s) = \mathcal{L}\,u(t)$ $\quad G(s) = \dfrac{Y(s)}{U(s)}$

$y \triangleq y(t)$, keluaran $\quad Y(s) = \mathcal{L}\,y(t)$

$x_1 \triangleq x(t)$

$x_2 \triangleq \dfrac{dx(t)}{dt} = \dot{x}_1 \longrightarrow \dot{x}_1 = x_2$

$x_3 \triangleq \dfrac{d^2x(t)}{dt^2} = \dot{x}_2 \longrightarrow \dot{x}_2 = x_3$

$\vdots$

$x_{n-2} \triangleq \dfrac{d^{n-3}x(t)}{dt^{n-3}} = \dot{x}_{n-3} \longrightarrow \dot{x}_{n-3} = x_{n-2}$

$x_{n-1} \triangleq \dfrac{d^{n-2}x(t)}{dt^{n-2}} = \dot{x}_{n-2} \longrightarrow \dot{x}_{n-2} = x_{n-1}$

$x_n \triangleq \dfrac{d^{n-1}x(t)}{dt^{n-1}} = \dot{x}_{n-1} \longrightarrow \dot{x}_{n-1} = x_n$

$u(t) = a_n\dfrac{d^nx(t)}{dt^n} + a_{n-1}\dfrac{d^{n-1}x(t)}{dt^{n-1}} + a_{n-2}\dfrac{d^{n-2}x(t)}{dt^{n-2}} + \dots + a_1\dfrac{dx(t)}{dt} + a_0x(t)$

$u = a_n\dfrac{d^nx(t)}{dt^n} + a_{n-1}x_n + a_{n-2}x_{n-1} + \dots + a_1x_2 + a_0x_1$

$\dot{x}_n = \dfrac{d^nx(t)}{dt^n} = -\dfrac{1}{a_n}\left(a_0x_1 + a_1x_2 + a_2x_3 + \dots + a_{n-2}x_{n-1} + a_{n-1}x_n\right) + \dfrac{1}{a_n}u$

Persamaan Keadaan:

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\vdots$

$\dot{x}_{n-1} = x_n$

$\dot{x}_n = -\dfrac{a_0}{a_n}x_1 - \dfrac{a_1}{a_n}x_2 - \dfrac{a_2}{a_n}x_3 - \dots - \dfrac{a_{n-1}}{a_n}x_n + \dfrac{1}{a_n}u$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & -\frac{a_2}{a_n} & \cdots & -\frac{a_{n-1}}{a_n} \end{bmatrix}}_{A[n\times n]} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \frac{1}{a_n} \end{bmatrix}}_{B[n\times 1]} u$$

* Pers. Keluaran.

Dari pers. (1):

$Y(s) = b_ms^mX(s) + b_{m-1}s^{m-1}X(s) + \dots + b_1sX(s) + b_0X(s)$

Transf. Laplace Inverse:

$y(t) = b_m\dfrac{d^mx(t)}{dt^m} + b_{m-1}\dfrac{d^{m-1}x(t)}{dt^{m-1}} + \dots + b_1\dfrac{dx(t)}{dt} + b_0x(t)$

$y = b_0x_1 + b_1x_2 + \dots + b_{m-1}x_m + b_mx_{m+1}$

$$y = \underbrace{\begin{bmatrix} b_0 & b_1 & b_2 & \cdots & b_{m-1} & b_m & 0 & 0 \end{bmatrix}}_{C[1\times n]} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \\ x_{m+1} \\ \vdots \\ x_n \end{bmatrix} + \underbrace{[\emptyset]}_{D[1\times 1]} u$$

Jika $m = n-1$ maka matrix C terisi penuh

Semua Model Nisbah Alih dapat diubah menjadi model Ruang Keadaan

$U(s) \to \boxed{G(s)} \to Y(s) \Rightarrow u \to \boxed{\begin{matrix}\dot{x} = Ax + Bu \\ y = Cx + Du\end{matrix}} \to y$ (SISO)

Bentuk Umum Model Nisbah Alih

$G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{b_ms^m + b_{m-1}s^{m-1} + b_{m-2}s^{m-2} + \dots + b_1s + b_0}{a_ns^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_1s + a_0}$

m, n : bilangan bulat, $\boxed{m \le n}$ tidak ada $m > n$

$a_0, a_1, a_2, \dots, a_{n-1}, a_n$ koefisien REAL

$b_0, b_1, b_2, \dots, b_{m-1}, b_m$ koefisien REAL

* Kasus $m < n$

* Kasus $m = n$

$X(s) = \mathcal{L}\,x(t)$

* Kasus $m < n$

$G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{(b_ms^m + b_{m-1}s^{m-1} + b_{m-2}s^{m-2} + \dots + b_1s + b_0)X(s)}{(a_ns^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_1s + a_0)X(s)}$

Jika $Y(s) = (b_ms^m + b_{m-1}s^{m-1} + b_{m-2}s^{m-2} + \dots + b_1s + b_0)X(s)$ → (1)

maka $U(s) = (a_ns^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_1s + a_0)X(s)$ → (2)

sebab : $\left[\dfrac{k}{m} = \dfrac{1}{2}\right] \longrightarrow$ [jika $k = 1$, maka $m = 2$]

Jadi

(2) $\longrightarrow$ $U(s) = a_ns^nX(s) + a_{n-1}s^{n-1}X(s) + a_{n-2}s^{n-2}X(s) + \dots + a_1sX(s) + a_0X(s)$

Trans. Laplace Inverse.

$u(t) = a_n\dfrac{d^nx(t)}{dt^n} + a_{n-1}\dfrac{d^{n-1}x(t)}{dt^{n-1}} + a_{n-2}\dfrac{d^{n-2}x(t)}{dt^{n-2}} + \dots + a_1\dfrac{dx(t)}{dt} + a_0x(t)$

Penentuan peubah keadaan

state assignment

Catatan: state assignment² bisa bermacam², karena itu model Ruang Keadaan yang dihasilkan bisa ber-macam² pula.

Contoh: Tentukan model Ruang Keadaan dengan matrix A berbentuk Matrix Kawanan Jordan (Jordan Companion) dan model Nisbah Alih:

$$G(s) = \frac{s^5 + 3s^3 + 5s + 1}{2s^8 + 4s^6 + 6s^4 + 8s^2 + 10}$$

$m = 5 \qquad n = 8 \qquad$ Kasus $m < n$

$b_0 = 1 \qquad a_0 = 10$

$b_1 = 5 \qquad a_1 = 0$

$b_2 = 0 \qquad a_2 = 8$

$b_3 = 3 \qquad a_3 = 0$

$b_4 = 0 \qquad a_4 = 6$

$b_5 = 1 \qquad a_5 = 0$

$a_6 = 4$

$a_7 = 0$

$a_8 = 2$

$$\begin{bmatrix} \dot{x} = Ax + Bu \\ y = Cx + Du \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_7 \\ x_8 \end{bmatrix}$$

$\underset{[1\times 8]}{C} = [1\ 5\ 0\ 3\ 0\ 1\ 0\ 0]$ ($b_0\ b_1\ b_2\ b_3\ b_4\ b_5$)

$\underset{[1\times 1]}{D} = [\emptyset]$

$$\underset{[8\times 1]}{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

$$\underset{[8\times 8]}{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{10}{2} & 0 & -\frac{8}{2} & 0 & -\frac{6}{2} & 0 & -\frac{4}{2} & 0 \end{bmatrix}$$

* Solusi Persamaan Keadaan

$$\boxed{\frac{dx(t)}{dt} = \dot{x} = Ax + Bu} \quad , \; x(0)$$

solusi $x(t) = \ldots\ldots ?$

Mulai dengan kasus "skalar", tanpa masukan u:

$$\frac{dx(t)}{dt} = \dot{x} = a\,x(t) \quad , \quad x(0) = x_0$$

$$\int \frac{dx(t)}{x(t)} = \int a\,dt = a\int dt = at$$

$$\ln\left[x(t)\right] = at + K$$

$${}^{e}\log\left[x(t)\right] = at + K$$

$$x(t) = e^{(at+K)}$$

$$x(t) = e^{K} \cdot e^{at}$$

$$= A e^{at}$$

$$t = 0 \rightarrow x(0) = A e^{0} = A$$

$$x_0 = A$$

Solusi: $\boxed{x(t) = x_0 e^{at}}$

check: $\dfrac{dx(t)}{dt} = a x_0 e^{at} = a\,x(t)$

checked

$$\int \frac{dx}{x} = \ln x$$

$$\int x\,dx = \frac{1}{2}x^2$$

$$\int dx = x$$

$$\int x^2\,dx = \frac{1}{3}x^3$$

$$\int x^3\,dx = \frac{1}{4}x^4$$

$$\int x^{-2}\,dx = -x^{-1} = -\frac{1}{x}$$

$$\int x^{-1}\,dx = \int \frac{dx}{x} = \ln x$$

$$\int x^{-3}\,dx = -\frac{1}{2}x^{-2}$$

$${}^{a}\log b = c$$

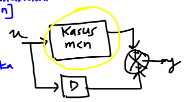
$$b = a^{c}$$

* Kasus $m=n$

Matrix A dan B tidak masalah

Matrix $C\,[1\times m] \rightarrow C\,[1\times n]$

b_0, b_1, b_2, …, b_{m-1}, b_m : $(m+1)$ angka



Contoh: $G(s) = \dfrac{s^8 + 3s^3 + 5s + 1}{2s^8 + 4s^6 + 6s^4 + 8s^2 + 10}$ → A & B sama dengan contoh sebelumnya

Pembagian:

$$2s^8 + 4s^6 + 6s^4 + 8s^2 + 10 \;\Big)\; s^8 + 3s^3 + 5s + 1 \;\;=\;\; 0{,}5$$

$$s^8 + 2s^6 + 3s^4 + 4s^2 + 5 \quad -$$

$$-2s^6 - 3s^4 + 3s^3 - 4s^2 + 5s - 4$$

$$G(s) = \frac{-2s^6 - 3s^4 + 3s^3 - 4s^2 + 5s - 4}{2s^8 + 4s^6 + 6s^4 + 8s^2 + 10} + 0{,}5$$

Kasus $m < n$ $\qquad$ [D]

$$\frac{8}{3} = \frac{2}{3} + 2$$

$$\underset{[1\times 8]}{C} = \begin{bmatrix} -4 & 5 & -4 & 3 & -3 & 0 & -2 & 0 \end{bmatrix}$$

$$\underset{[1\times 1]}{D} = [0{,}5]$$

Misalnya:

$$x(t) = x_0\left(1 + at + \frac{1}{2}a^2t^2 + \frac{1}{6}a^3t^3 + \frac{1}{24}a^4t^4\right)$$

$$\frac{dx(t)}{dt} = x_0\left(a + a^2t + \frac{1}{2}a^3t^2 + \frac{1}{6}a^4t^3 + \ldots\right)$$

$$= a x_0\left(1 + at + \frac{1}{2}a^2t^2 + \frac{1}{6}a^3t^3 + \ldots\right)$$

$$= a\,x(t)$$

maka jelaslah: $\boxed{e^{at} = 1 + at + \frac{1}{2}a^2t^2 + \frac{1}{6}a^3t^3 + \ldots}$

$$e^1 = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \ldots \approx 2{,}718\ldots$$

*Kasus MATRIKS

$\dot{x} = Ax \qquad x = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \qquad A\,[n \times n] \qquad x_0 = \begin{bmatrix} x_1(0) \\ x_2(0) \\ \vdots \\ x_n(0) \end{bmatrix}$

$$x(t) = [I + At + \tfrac{1}{2}A^2t^2 + \tfrac{1}{6}A^3t^3 + \tfrac{1}{24}A^4t^4 + \quad)x_0$$

$$\frac{dx(t)}{dt} = [A + A^2t + \tfrac{1}{2}A^3t^2 + \tfrac{1}{6}A^4t^3 + \cdots)x_0$$

$$= A[I + At + \tfrac{1}{2}A^2t^2 + \tfrac{1}{6}A^3t^3 + \cdots)x_0$$

$$\dot{x} = Ax(t)$$

$$\boxed{I + At + \tfrac{1}{2}A^2t^2 + \tfrac{1}{6}A^3t^3 + \cdots \equiv e^{At}} \rightarrow [n \times n]$$

Solusi $\dot{x} = Ax \rightarrow x(t) = e^{At}x_0 \longrightarrow$ solusi khusus

Bagaimana solusi dari $\dot{x} = Ax + Bu$

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$

Misalnya $x(t) = e^{At}c(t)$

maka $\dfrac{dx(t)}{dt} = \underbrace{Ae^{At}c(t)} + e^{At}\dfrac{dc(t)}{dt} = Ax(t) + Bu(t)$

$$\cancel{Ax(t)} + e^{At}\frac{dc(t)}{dt} = \cancel{Ax(t)} + Bu(t)$$

$$e^{At}\frac{dc(t)}{dt} = Bu(t)$$

$$\underbrace{[e^{At}]^{-1}[e^{At}]}_{I} \cdot \frac{dc(t)}{dt} = [e^{At}]^{-1}Bu(t)$$

$$\frac{dc(t)}{dt} = [e^{At}]^{-1}Bu(t)$$

$$\int dc(t) = \int [e^{At}]^{-1}Bu(t)\,dt$$

$$c(t) = \int [e^{At}]^{-1}Bu(t)\,dt$$

$$x(t) = e^{At}c(t)$$

$$= e^{At}\int [e^{At}]^{-1}Bu(t)\,dt \longrightarrow \text{solusi umum}$$

Karena kerumitan SOLUSI ANALITIK dari model Ruang Keadaan maka orang lebih suka mencari SOLUSI NUMERIK menggunakan SIMULASI

SOLUSI :

$$\boxed{x(t) = e^{At}x_0 + e^{At}\int [e^{At}]^{-1}Bu(t)\,dt}$$

di-substitusi ke persamaan keluaran

$$y = Cx + Du$$

Keluaran :

$$\boxed{y(t) = Cx(t) + Du(t)}$$

next ! Transformasi SIMILARITAS

$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ $\bar{A} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

Nilai eigen λ dapat diperoleh dari: $|\lambda I - A| = 0$

Untuk matrix A.

$$\lambda I - A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & -1 \\ -1 & \lambda - 1 \end{bmatrix}$$

$$\det[\lambda I - A] = \lambda(\lambda - 1) - 1 = \lambda^2 - \lambda - 1 = 0$$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1}{2} \pm \frac{1}{2}\sqrt{5}$$

untuk matrix $\bar{A}$:

$$[\lambda I - \bar{A}] = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} \lambda + \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \lambda - \frac{3}{2} \end{bmatrix}$$

$$\rightarrow \det[\lambda I - \bar{A}] = (\lambda + \tfrac{1}{2})(\lambda - \tfrac{3}{2}) - \tfrac{1}{4} = \lambda^2 - \lambda - \tfrac{3}{4} - \tfrac{1}{4} = \lambda^2 - \lambda - 1$$

$$\det[\lambda I - \bar{A}] = 0 \rightarrow \lambda^2 - \lambda - 1 = 0$$

Contoh

Suatu sistem dimodelkan dengan model Ruang Keadaan

$\dot{x} = Ax + Bu$ $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$y = Cx + Du$ $C = [1 \ \ 0]$ $D = [0]$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

dengan matrix $T = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ ubahlah model sehingga peubah keadaannya

$\bar{x} = Tx$

$$\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = Tx = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_1 + 2x_2 \end{bmatrix}$$

Matrix T invertible $\det(T) = 2 \neq 0$

$$T^{-1} = \frac{adj(T)}{|T|} = \frac{\begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}}{2} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\bar{B} = TB = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\bar{A} = TAT^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 3/2 \end{bmatrix}$$

$$\bar{C} = CT^{-1} = [1 \ \ 0] \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} = [1 \ \ 0] \qquad \bar{D} = D = [0]$$

$$T^{-1}\bar{A}T = A$$

$$TT^{-1}\bar{A}TT^{-1} = TAT^{-1}$$

$$\boxed{\bar{A} = TAT^{-1}}$$

$$T^{-1}\bar{B} = B$$

$$TT^{-1}\bar{B} = TB$$

$$\boxed{\bar{B} = TB}$$

$$\bar{C}T = C \rightarrow \bar{C}TT^{-1} = CT^{-1}$$

$$\boxed{\bar{C} = CT^{-1}}$$

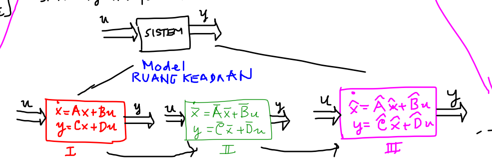
$$\boxed{\bar{C}T = C}$$

$$\boxed{\bar{D} = D}$$

$$y = \bar{C}Tx + \bar{D}u = Cx + Du$$

* Transformasi Similaritas

Similarity Transformation



Dari sistem yang sama dapat disusun banyak model Ruang Keadaan yang berbeda-beda, tapi sesungguhnya sama (SIMILAR), tergantung pada pemilihan peubah keadaan, $x, \bar{x}, \hat{x}$, dst

Transformasi dari model I ke model II, ke model III dst dengan mengubah peubah keadaan $x \rightarrow \bar{x} \rightarrow \hat{x} \rightarrow \ldots$ disebut Transformasi Similaritas.

Misalnya, Model $\dot{x} = Ax + Bu$

$y = Cx + Du$

akan diubah menjadi.

Model $\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u$

$y = \bar{C}\bar{x} + \bar{D}u$

dengan $\bar{x} = Tx$, T suatu matrix yang invertible dengan inverse. T^{-1}

Note! Matrix yang punya inverse:

- * Determinan-nya $\neq 0$
- * Baris/Kolom bebas linier
- * Non-singular.

$$T\dot{x} = \bar{A}Tx + \bar{B}u$$

$$y = \bar{C}Tx + \bar{D}u$$

$$T^{-1}T\dot{x} = T^{-1}\bar{A}Tx + T^{-1}\bar{B}u$$

$$\dot{x} = T^{-1}\bar{A}Tx + T^{-1}\bar{B}u = Ax + Bu \rightarrow \boxed{\begin{matrix} T^{-1}\bar{A}T = A \\ T^{-1}\bar{B} = B \end{matrix}}$$

$a^{-1} \cdot a = 1$

$\frac{1}{5} \cdot 5 = 1$

$A^{-1} \cdot A = I$

Persamaan.

$$\det[\lambda I - A] = 0$$

disebut pers. karakteristik

Jadi matrix A dan $\bar{A}$ mempunyai pers. karakteristik yg. sama!

$$\lambda^2 - \lambda - 1 = 0$$

berarti nilai eigen kedua matrix itu sama

Kesimpulan.

Transformasi Similaritas tidak mengubah pers. karakteristik dari matrix A, sehingga nilai eigen-nya tetap

Dari kesimpulan ini, dapat dipastikan bahwa untuk setiap matrix A dari model ruang keadaan dapat diperoleh matrix yang "similar" dalam format.

$$\bar{A} = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \lambda_n \end{bmatrix}_{[n \times n]}$$

dengan $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ adalah nilai eigen matrix A yaitu nilai eigen matrix $\bar{A}$ sendiri.

Untuk matrix A diagonal

$$A = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

maka
$$e^{At} = \begin{bmatrix} e^{\lambda_1 t} & & & 0 \\ & e^{\lambda_2 t} & & \\ & & \ddots & \\ 0 & & & e^{\lambda_n t} \end{bmatrix}$$

sehingga untuk $u = 0$ (tanpa input)

$$x(t) = e^{At} x_0$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} = \begin{bmatrix} e^{\lambda_1 t} x_1(0) \\ e^{\lambda_2 t} x_2(0) \\ \vdots \\ e^{\lambda_n t} x_n(0) \end{bmatrix}$$

Jika $\lambda_1 = a_1 + jb_1$ (bilangan kompleks), maka

$$e^{\lambda_1 t} = e^{(a_1 + jb_1)t} = e^{a_1 t} \underbrace{e^{jb_1 t}}_{\text{osilasi}}$$

TIDAK STABIL $\begin{cases} a_1 = 0 \rightarrow e^{a_1 t} = 1 \rightarrow \text{osilasi tetap} \\ a_1 > 0 \rightarrow e^{a_1 t} \rightarrow \text{membesar}, t \rightarrow \infty \end{cases}$

STABIL $\leftarrow a_1 < 0 \rightarrow e^{a_1 t} \rightarrow$ mengecil, $t \rightarrow \infty$

Suatu sistem yang dimodelkan dengan model ruang keadaan akan STABIL jika dan hanya jika nilai eigen matrix A SEMUA berada di sebelah KIRI sumbu khayal pada bidang kompleks.

akar-akar pers. karakteristik

Jika ada nilai eigen matrix A yang ada pada sumbu khayal atau di sebelah kanannya, walau pun hanya satu, maka sistem TIDAK STABIL.

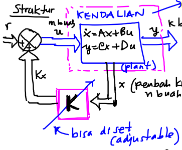
next: Keterkendalian & Keteramatan

Sistem Kendali Umpan-balik Peubah Keadaan

* Umpan Balik Peubah Keadaan

State Variable Feedback

Struktur



$u\,[m \times 1]$, $r\,[m \times 1]$, $Kx\,[m \times 1]$, $x\,[n \times 1]$, $A = [n \times n]$

$K\,[m \times n]$ Gain Matrix $B = [n \times m]$, $K = [m \times n]$, $BK = [n \times n]$

$u = r + Kx \longrightarrow \dot{x} = Ax + Bu$

$= Ax + B[r + Kx]$

$= Ax + BKx + Br$

$= [A + BK]x + Br$

$= \bar{A}x + Br$

$\boxed{\bar{A} = A + BK}$ set

Agar sistem kendali ini STABIL maka nilai-eigen matrix $\bar{A}$ semua harus berada di sebelah kiri sb. khayal pada bidang komplex

K adjustment $\longrightarrow$ pole placement eigen-value placement

* Contoh:

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ $C = [3\ \ 4]$

tentukan gain matrix K agar sistem kendali stabil dengan nilai-eigen matrix $\bar{A}$ pada $\lambda_1 = -2 + j$, $\lambda_2 = -2 - j$

Misalnya $K = [k_1\ \ k_2]$ $[1 \times 2]$ $m \times n$

$\bar{A} = [A + BK] = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}[k_1\ \ k_2] = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 2k_1 & 2k_2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 2k_1 & 2k_2 \end{bmatrix}$

$\lambda I - \bar{A} = \begin{bmatrix} \lambda & -1 \\ -2k_1 & \lambda - 2k_2 \end{bmatrix}$

$\det[\lambda I - \bar{A}] = 0$

$\lambda[\lambda - 2k_2] - 2k_1 = 0$

$\lambda^2 - 2k_2\lambda - 2k_1 = 0$

$(\lambda + 2 - j)(\lambda + 2 + j) = 0$

$\lambda^2 + 4\lambda + 1 = 0$

$-2k_1 = 1$

$k_1 = -\frac{1}{2}$

$-2k_2 = 4$

$k_2 = -2$

$K = \left[-\frac{1}{2}\ \ -2\right]$

Contoh. Kendalian (PLANT) $\dot{x} = Ax + Bu$, $y = Cx$ $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ $C = [3\ \ 4]$

SISO, 2 peubah keadaan

* Stabilkah kendalian di atas?

* Apakah sepenuhnya terkendali?

* Apakah sepenuhnya teramati?

* Kestabilan

Nilai eigen matrix A λ

$\det[\lambda I - A] = 0$ $[\lambda I - A] = \begin{bmatrix} \lambda & -1 \\ 0 & \lambda \end{bmatrix}$

$\lambda^2 = 0 \rightarrow \lambda_1 = 0,\ \lambda_2 = 0$ tidak berada di sebelah kiri sumbu khayal

Jadi kendalian ini TIDAK STABIL

* Keterkendalian

Matrix Keterkendalian P

$P = [B\ AB]$

$= \left[\begin{bmatrix} 0 \\ 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix}\right]$

$= \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ $\det[P] = -4 \neq 0$

$\text{rank}[P] = 2 = n$

Kendalian sepenuhnya terkendali $\rightarrow$ bisa distabilkan

* Keteramatan

Matrix Keteramatan Q

$Q = [C^T\ A^TC^T]$

$= \left[\begin{bmatrix} 3 \\ 4 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix}\right]$

$= \begin{bmatrix} 3 & 0 \\ 4 & 3 \end{bmatrix} \longrightarrow \det[Q] = 9 \neq 0$, $\text{rank}[Q] = 2 = n$

Kendalian sepenuhnya teramati; bisa di-stabil-kan tanpa harus merancang OBSERVER (pengamat)

Untuk memeriksa Keterkendalian dan Keteramatan (Controllability & Observability) suatu kendalian (PLANT) maka bisa digunakan matrix keterkendalian P dan matrix Keteramatan Q

$P = [B\ \ AB\ \ A^2B\ \ \ldots\ \ A^{n-1}B]$ $A[n \times n]$, $B[n \times m]$, m = jumlah masukan

$Q = [C^T\ \ A^TC^T\ \ [A^T]^2C^T\ \ldots\ \ [A^T]^{n-1}C^T]$ $C[k \times n]$, k = jumlah keluaran

* Jika matrix P dan Q bujur sangkar $[n \times n]$, maka sistem sepenuhnya terkendali dan teramati jika determinan matrix P dan Q tidak sama dengan nol.

Karena matrix $[n \times n]$ yang determinannya tidak sama dengan nol:

- * rank-nya = n (penuh)
- * semua baris/kolom bebas linier
- * non-singular
- * mempunyai inverse

* Jika matrix P dan Q bukan matrix bujursangkar, maka:

- Sistem sepenuhnya terkendali jika rank(P) = n, P full rank

∃ ada n baris/kolom yang bebas linier

nxn

bukan bujursangkar

bebas linier

$[\ ] \neq \alpha_1[\ ] + \alpha_2[\ ] + \ldots$

* Sistem sepenuhnya teramati jika rank(Q) = n, Q full rank

Rencana

- * 21/11 → Selesaikan MATERI
- * 28/11 → TEST I (materi dari RH7)
  - ↳ Petunjuk Praktikum
- * 28/11 – 05/12 Pembagian INDIVIDU KELOMPOK PRAKTIKUM
- * 05/12 LSKI Praktikum PERDANA
- Akhir semester → TEST II (materi = TEST I)

* Keterkendalian dan Keteramatan

Controllability and Observability

Tujuan utama pengendalian adalah men-STABIL-kan sistem atau kendalian

Contoh. $\dot{x} = Ax + Bu$, $y = Cx + Du$ SISO

$A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $C = [1\ 1\ 0\ 0]$ $D = [0]$

Pers Keadaan:

$\dot{x}_1 = -x_1 + u$

$\dot{x}_2 = -2x_2$

$\dot{x}_3 = -3x_3 + u$

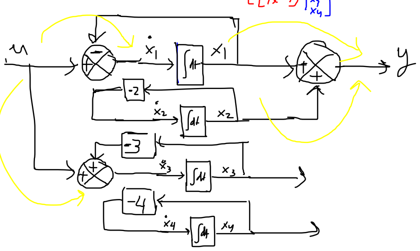
$\dot{x}_4 = -4x_4$

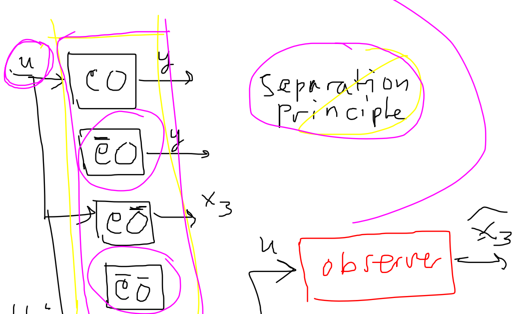
$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} u$

$A[4 \times 4]$ $B[4 \times 1]$

Pers Keluaran: $y = x_1 + x_2$

$y = [1\ 1\ 0\ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ $C[1 \times 4]$





Separation Principle

Jika $\bar{C}O$ dan $\bar{C}\bar{O}$ tidak stabil maka sistem akan un-stabilizable (tidak mungkin di-stabil-kan dengan desain pengendali apa pun)

Jika $C\bar{O}$ yang tidak stabil, maka sistem mungkin di-stabilkan dengan merancang suatu sistem pengamat (OBSERVER)

WRAP UP

* Model Ruang Keadaan
* SS2TF   SISO
* TF2SS
* Solusi $\dot{x} = Ax + Bu$
* Transf. SIMILARITAS
* Kestabilan (nilai-eigen A)
* Keterkendalian & Keteramatan
* Umpan Balik Peubah Keadaan