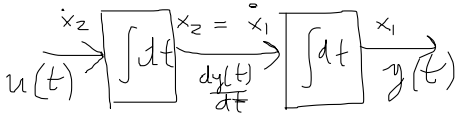
Contoh: "Double Integrator"



State assignment:

$x_1 \triangleq y(t) \rightarrow \dot{x}_1 = \frac{dy(t)}{dt}$

$x_2 \triangleq \frac{dy(t)}{dt} \quad \dot{x}_1 = x_2$

$u \triangleq \dot{x}_2 \quad \dot{x}_2 = u$

$y = y(t) = x_1$

Pers. keadaan

$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

Pers. keluaran: $A[2\times2]$ $B[2\times1]$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$

$C[1\times2]$ $D[1\times1]$

Model Ruang Keadaan dari "Double Integrator"

Model Nisbah Alih: $[sI-A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$

$[sI-A]^{-1} = \frac{\text{adj}[sI-A]}{|sI-A|} = \frac{\begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix}}{s^2} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix}$

$G(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0 = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0 = \frac{1}{s^2}$

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>> help ss2tf | MATLAB Functions
>> help tf2ss | Control Tool Box
"State Space" to "Transfer Function"
"Transfer Function to "State Space"
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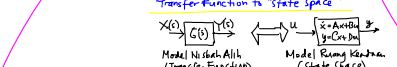

Semua Model Nisbah Alih dapat dibuat Model Ruang Keadaan, tapi tidak sebaliknya

* ss2tf Model Ruang Keadaan yang dapat diubah menjadi model Nisbah Alih hanya model sistem SISO (single input single output)

$\Rightarrow A[n\times n]$, $B[n\times1]$, $C[1\times n]$, $D[1\times1]$

$m=1$ dan $k=1$ (SISO)

$\dot{x} = Ax + Bu \quad u \triangleq u(t)$ single input

$sIX(s) - AX(s) = BU(s)$

$[sI-A]\dot{X}(s) = BU(s)$

$\underbrace{[sI-A]^{-1}[sI-A]}_{I} X(s) = [sI-A]^{-1} BU(s)$

$X(s) = [sI-A]^{-1} BU(s)$

$y = Cx + Du$

Transf. Laplace:

$Y(s) = CX(s) + DU(s)$

$= C[sI-A]^{-1}BU(s) + DU(s)$

$= \left[C[sI-A]^{-1}B + D\right] U(s)$

Transfer Function:

$$G(s) = \left[C[sI-A]^{-1}B + D\right]$$

Khusus SISO

$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} = A[n\times n] \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} + B[n\times1]\, u(t)$

Transf. Laplace:

$\begin{bmatrix} sX_1(s) \\ sX_2(s) \\ \vdots \\ sX_n(s) \end{bmatrix} = A \begin{bmatrix} X_1(s) \\ X_2(s) \\ \vdots \\ X_3(s) \end{bmatrix} + BU(s)$

$\underbrace{\begin{bmatrix} sX_1(s) \\ sX_2(s) \\ \vdots \\ sX_n(s) \end{bmatrix}}_{[n\times1]} - \underbrace{A}_{[n\times n]} \underbrace{\begin{bmatrix} X_1(s) \\ X_2(s) \\ \vdots \\ X_n(s) \end{bmatrix}}_{[n\times1]} = \underbrace{BU(s)}_{[n\times1]\ [1\times1]}$

$sIX(s) - AX(s) = BU(s)$

$I = \begin{bmatrix} 1 & 0 \\ & 1 \\ 0 & \ddots \\ & & 1 \end{bmatrix}$

$sI = \underbrace{\begin{bmatrix} s & 0 \\ & s \\ 0 & \ddots \\ & & s \end{bmatrix}}_{[n\times n]}$