Sebagai contoh, berikut ini adalah penentuan peubah keadaan yang menghasilkan bentuk matrix A khusus yang disebut "Jordan Companion matrix"

$u \triangleq u(t)$, masukan $U(s) = \mathcal{L}[u(t)]$ $G(s) = \frac{Y(s)}{U(s)}$

$y \triangleq y(t)$, keluaran $Y(s) = \mathcal{L}[y(t)]$

$x_1 \triangleq y(t)$

$x_2 \triangleq \frac{dx(t)}{dt} = \dot{x}_1 \rightarrow \dot{x}_1 = x_2$

$x_3 \triangleq \frac{d^2x(t)}{dt^2} = \dot{x}_2 \rightarrow \dot{x}_2 = x_3$

$\vdots$

$x_{n-2} \triangleq \frac{d^{n-3}x(t)}{dt^{n-3}} = \dot{x}_{n-3} \rightarrow \dot{x}_{n-3} = x_{n-2}$

$x_{n-1} \triangleq \frac{d^{n-2}x(t)}{dt^{n-2}} = \dot{x}_{n-2} \rightarrow \dot{x}_{n-2} = x_{n-1}$

$x_n \triangleq \frac{d^{n-1}x(t)}{dt^{n-1}} = \dot{x}_{n-1} \rightarrow \dot{x}_{n-1} = x_n$

$u(t) = a_n \frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + a_{n-2}\frac{d^{n-2}y(t)}{dt^{n-2}} + \cdots + a_1\frac{dx(t)}{dt} + a_0 x(t)$

$u = a_n \frac{d^n x(t)}{dt^n} + a_{n-1}x_n + a_{n-2}x_{n-1} + \cdots + a_1 x_2 + a_0 x_1$

$\dot{x}_n = \frac{d^n x(t)}{dt^n} = \frac{1}{a_n}\left(a_0 x_1 + a_1 x_2 + a_2 x_3 + \cdots + a_{n-2}x_{n-1} + a_{n-1}x_n\right) + \frac{1}{a_n}u$

Contoh. Tentukan model ruang keadaan dengan matrix A berbentuk Matrix Kawanan Jordan (Jordan Companion) dan model Nisbah Alih:

$$G(s) = \frac{s^5 + 3s^3 + 5s + 1}{2s^8 + 4s^5 + 6s^4 + 8s^2 + 10}$$

$m = 5$, $n = 8$, kasus $m < n$

$b_0 = 1$, $a_0 = 10$

$b_1 = 5$, $a_1 = 0$

$b_2 = 0$, $a_2 = 8$

$b_3 = 3$, $a_3 < 0$

$b_4 = 0$, $a_4 < 6$

$b_5 = 1$, $a_5 = 0$

$a_6 = 4$

$a_7 = 0$

$a_8 = 2$

$$\begin{bmatrix}\dot{x} = Ax + Bu \\ y = Cx + Du\end{bmatrix}$$

$$x = \begin{bmatrix}x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_8\end{bmatrix}$$

$C_{[1\times8]} = [\,\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\,]$

$D_{[1\times1]} = [\emptyset]$

$$B_{[8\times1]} = \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\\frac{1}{2}\end{bmatrix}$$

$$A_{[8\times8]} = \begin{bmatrix}0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\\-\frac{10}{2}&0&-\frac{8}{2}&0&-\frac{6}{2}&0&-\frac{4}{2}&0\end{bmatrix}$$

Persamaan Kadar:

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\vdots$

$\dot{x}_{n-1} = x_n$

$\dot{x}_n = -\frac{a_0}{a_n}x_1 - \frac{a_1}{a_n}x_2 - \frac{a_2}{a_n}x_3 - \cdots - \frac{a_{n-1}}{a_n}x_n + \frac{1}{a_n}u$

$$\dot{x} = \begin{bmatrix}\dot{x}_1\\\dot{x}_2\\\vdots\\\dot{x}_{n-1}\\\dot{x}_n\end{bmatrix} = \begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\&&&\ddots&\\0&0&0&\cdots&1\\-\frac{a_0}{a_n}&-\frac{a_1}{a_n}&-\frac{a_2}{a_n}&\cdots&-\frac{a_{n-1}}{a_n}\end{bmatrix}\begin{bmatrix}x_1\\x_2\\\vdots\\x_{n-1}\\x_n\end{bmatrix} + \begin{bmatrix}0\\0\\\vdots\\0\\\frac{1}{a_n}\end{bmatrix}u$$

$A_{[n\times n]}$ $\quad B_{[n\times1]}$

Semua Model Nisbah Alih dapat diubah menjadi model Ruang Keadaan

Bentuk Umum Model Nisbah Alih

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + b_{m-2}s^{m-2} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s + a_0}$$

m, n : bilangan bulat, $m \le n$, tidak ada $m > n$

$a_0, a_1, a_2, \cdots, a_{n-1}, a_n$ koefisien REAL

$b_0, b_1, b_2, \cdots, b_{m-1}, b_m$ koefisien REAL

$X(s) = \mathcal{L}[x(t)]$

* Kasus $m < n$

* Kasus $m = n$

* Kasus $m < n$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\left(b_m s^m + b_{m-1}s^{m-1} + b_{m-2}s^{m-2} + \cdots + b_1 s + b_0\right)X(s)}{\left(a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s + a_0\right)X(s)}$$

Jika $Y(s) = \left(b_m s^m + b_{m-1}s^{m-1} + b_{m-2}s^{m-2} + \cdots + b_1 s + b_0\right)X(s)$ → (1)

maka $U(s) = \left(a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s + a_0\right)X(s)$ → (2)

* Pers. Keluaran.

Dari pers. (1):

$Y(s) = b_m s^m X(s) + b_{m-1}s^{m-1}X(s) + \cdots + b_1 s X(s) + b_0 X(s)$

Transf. Laplace Inverse:

$y(t) = b_m \frac{d^m x(t)}{dt^m} + b_{m-1}\frac{d^{m-1}x(t)}{dt^{m-1}} + \cdots + b_1\frac{dx(t)}{dt} + b_0 x(t)$

$y = b_0 x_1 + b_1 x_2 + \cdots + b_{m-1}x_m + b_m x_{m+1}$

$$y = \underbrace{[b_0 b_1 b_2 \cdots b_{m-1} b_m \cdots 0]}_{C_{[1\times n]}}\begin{bmatrix}x_1\\x_2\\\vdots\\x_m\\x_{m+1}\\\vdots\\x_n\end{bmatrix} + \underbrace{[\emptyset]}_{D_{[1\times1]}}u$$

Jika $m = n-1$ maka matrix C terisi penuh

Sebab: $\left[\frac{k}{m} = \frac{1}{2}\right] \rightarrow$ jika $k = 1$, maka $m = 2$

Jadi (2) → $U(s) = a_n s^n X(s) + a_{n-1}s^{n-1}X(s) + a_{n-2}s^{n-2}X(s) + \cdots + a_1 s X(s) + a_0 X(s)$

Trans. Laplace Inverse.

$$u(t) = a_n \frac{d^n x(t)}{dt^n} + a_{n-1}\frac{d^{n-1}x(t)}{dt^{n-1}} + a_{n-2}\frac{d^{n-2}x(t)}{dt^{n-2}} + \cdots + a_1\frac{dx(t)}{dt} + a_0 x(t)$$

Penentuan peubah keadaan

State assignment

Catatan: State assignment bisa ber-macam^2, karena itu model Ruang Keadaan yang dihasilkan bisa ber-macam^2 pula.