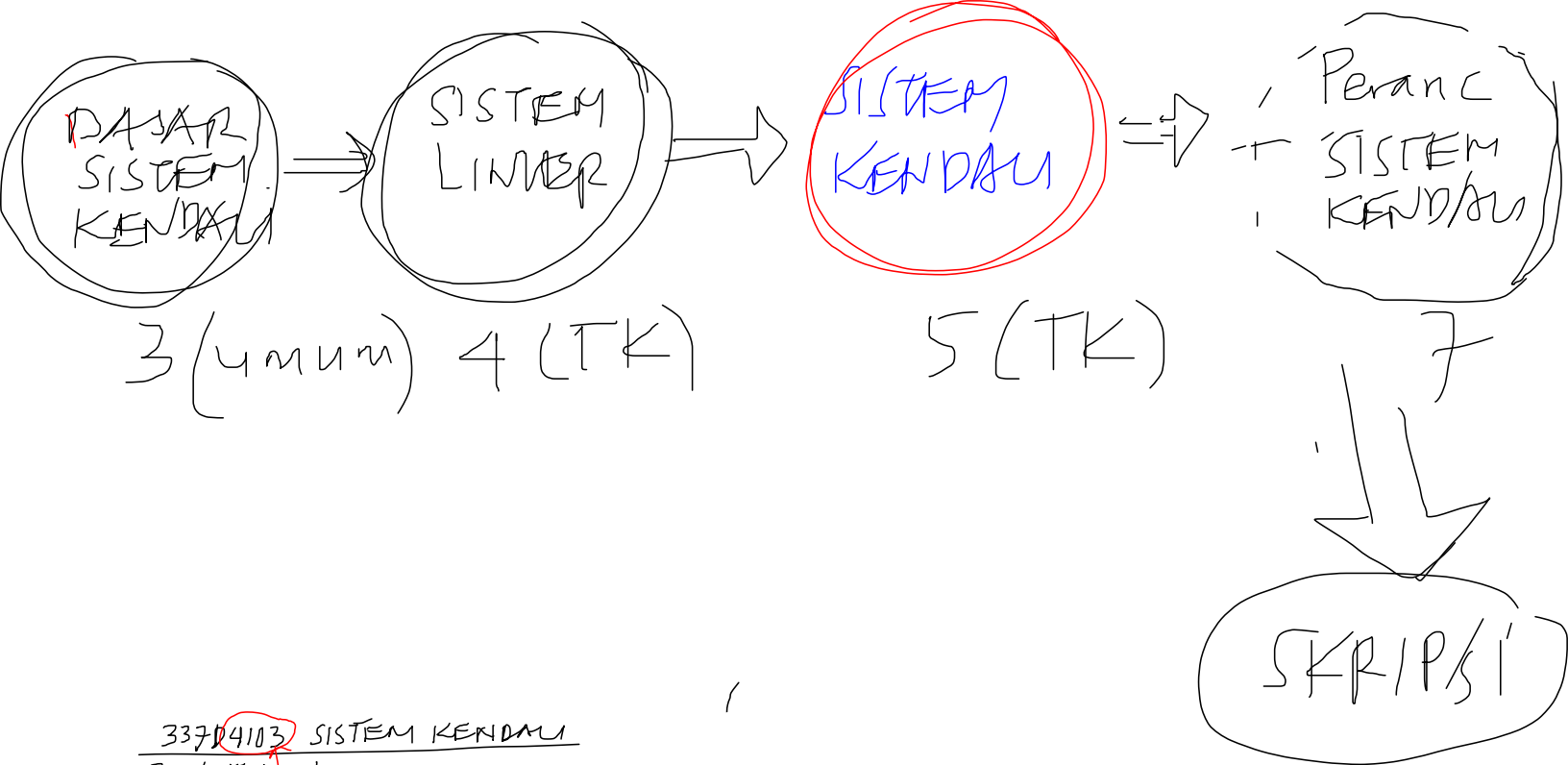




33704103 SISTEM KENDALI
Rinat Jadhav

3 SKS. * 1 SKS : pekan 1 s/d 8 RIZ
1 SKS : pekan 9 s/d 16 AEU
1 SKS : PRAKTIKUM.
- Praktikum Individu
- Praktikum Kelompok

3 SKS

Penilaian N.A = $RIZ + AEU + PRAKTIKUM$

3

← kurang lebih

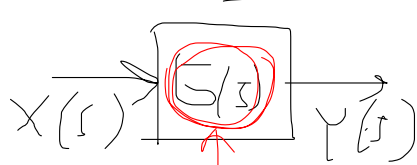
Link : <http://www.unhas.ac.id/rhiza/arsip/kuliah/>

RIZ : $\begin{bmatrix} \text{TEST 1 (pekan ke-8)} \\ \text{TEST 2 (pekan ke-16)} \end{bmatrix}$
→ Nilai RIZ : $\frac{\text{Test 1} + \text{Test 2}}{2}$

Silabus dapat dilihat di:

<http://www.unhas.ac.id/rhiza/arsip/kuliah/silabus-2008/>

* Hanya memperlihatkan hubungan Input-output saja, tidak memperlihatkan DINAMIKA INTERNAL sistem



$$G(s) = \frac{Y(s)}{X(s)}$$

* Model Ruang Keadaan bersifat lebih UMUM (general) dari Model Nisbah Alih: semua sistem yang dapat dimodelkan dengan model Nisbah Alih akan dapat dimodelkan dengan Model Ruang Keadaan, tapi belum tentu sebaliknya.

* Model Nisbah Alih hanya dapat memodelkan sistem linier. Model Ruang Keadaan dapat memodelkan sistem Tak Linier juga.

LTV LTI saja

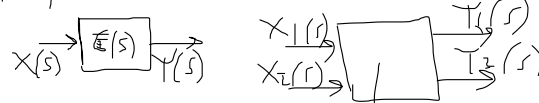
Apa kelebihan Model Nisbah Alih? Pratik & dan Sederhana

Model Ruang Keadaan

State Space Model

Kekurangan Model Nisbah-Alih (Transfer Function) :

* Hanya cocok untuk sistem "sederhana", SISO



4 (empat) Nisbah Alih

Model Ruang Keadaan bisa untuk sistem MIMO

$$\begin{cases} G_1(s) = \frac{Y_1(s)}{X_1(s)} \mid X_2(s) = 0 \\ G_2(s) = \frac{Y_2(s)}{X_1(s)} \mid X_2(s) = 0 \\ G_3(s) = \frac{Y_1(s)}{X_2(s)} \mid X_1(s) = 0 \\ G_4(s) = \frac{Y_2(s)}{X_2(s)} \mid X_1(s) = 0 \end{cases}$$

* Analisis dengan model Nisbah Alih lebih efektif untuk "pencil & paper", sulit untuk simulasi dengan komputer



Tabel Laplace

$$\mathcal{L}^{-1} \frac{1}{s+2} = \dots = e^{-2t}$$

$$\frac{1}{s+a} \rightarrow e^{-at}$$

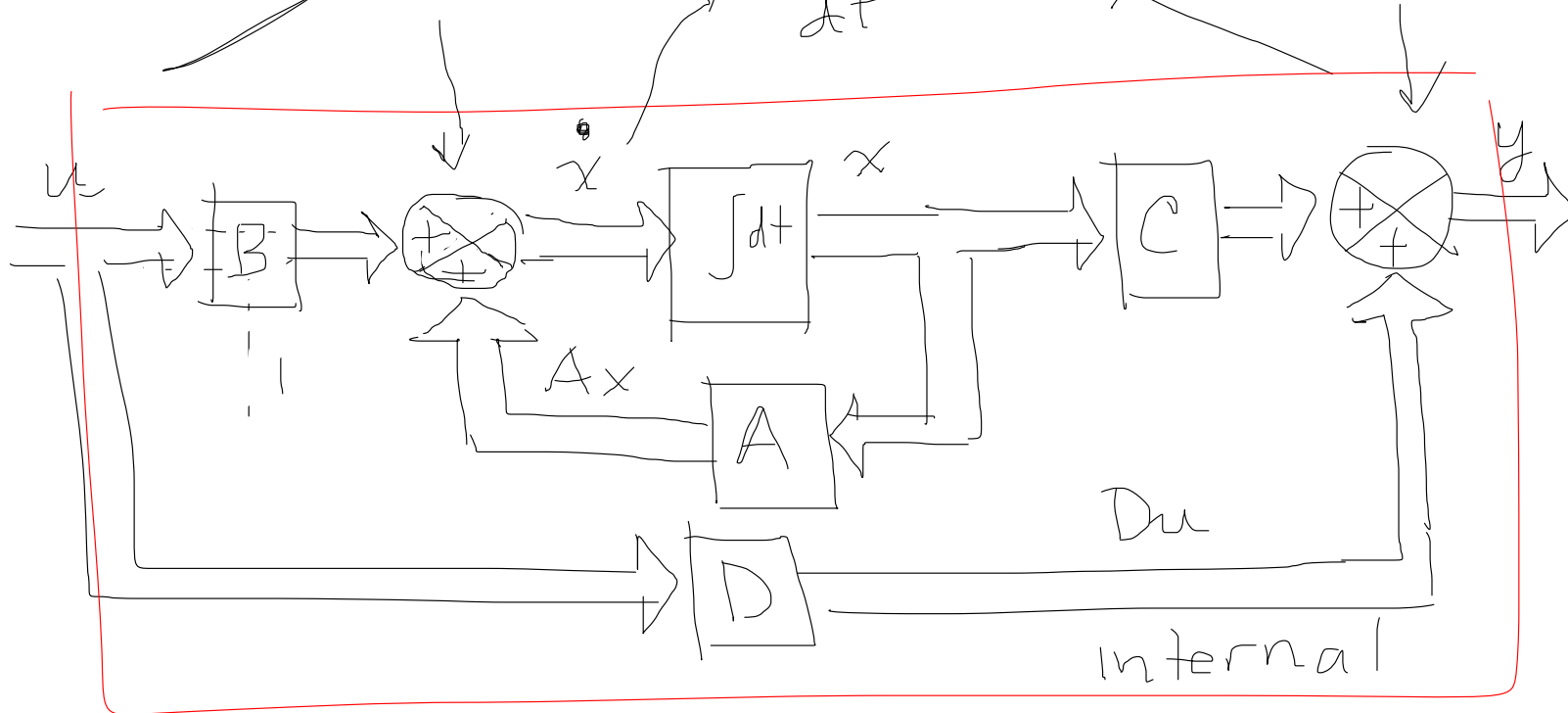
Model Ruang Keadaan (State space)



Bagan Kotak

$$\frac{dx(t)}{dt}$$

dinamika



Pertama:

Kedua:

(1) Persamaan (Masukan):

(2) Persamaan Keluaran:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

Aljabar Matriks (Aljabar Linier)

$$\dot{x} = Ax + Bu$$

Dimensi: $A [n \times n]$, $B [n \times m]$, $x [n \times 1]$, $u [m \times 1]$

Definisi:

$$u \triangleq u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix} \text{ vektor kolom } [m \times 1]$$

masuk INPUT

$$y \triangleq y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_k(t) \end{bmatrix} \text{ vektor kolom } [k \times 1]$$

keluaran OUTPUT

$$x \triangleq x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \text{ vektor kolom } [n \times 1]$$

peubah keadaan (state variable)

$$\dot{x} \triangleq \frac{dx(t)}{dt} = \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \vdots \\ \frac{dx_n(t)}{dt} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} \text{ vektor kolom } [n \times 1]$$

Contoh: Suatu sistem mempunyai 3 masukan, 4 keluaran dan 5 peubah-keadaan. Tentukan dimensi matriks A, B, C dan D

Jawab: $m=3$, $n=5$, $k=4$

$$A [n \times n] \rightarrow A [5 \times 5]$$

$$B [n \times m] \rightarrow B [5 \times 3]$$

$$C [k \times n] \rightarrow C [4 \times 5]$$

$$D [k \times m] \rightarrow D [4 \times 3]$$

Catatan:

* Jika matriks A, B, C dan D semuanya berisi KONSTANTA, maka sistem linier yang time-invariant (LTI)

* Jika di antara matriks A, B, C dan D ada yang berisi peubah fungsi waktu t, maka sistem

linier time-varying (LTV)

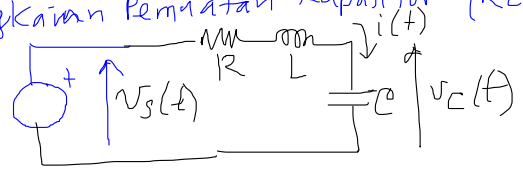
* Selain itu sistem tak linier

* Model Nisbah Ahh hanya dapat memodelkan sistem LTI saja

Next Contoh

Next
 $ss2+ff$
 $tf2ss$

* Contoh Elektrik Rangkaian Pematan Kapasitor (RLC seri)



Susunlah Model Ruang Keadaan.
 $\dot{x} = Ax + Bu$
 $y = Cx + Du$
 dengan penurukan (state assignment)

* Contoh Mekanik :
 Spring-Mass-Damper System
 Susunlah Model Ruang Keadaan dengan x dan $F(t)$ sebagai masukan, $m=1$ dengan $y = x$ sebagai keluaran $k=1$
 $\dot{x} = Ax + Bu$
 $y = Cx + Du$
 * Dimensi matrix $A [2 \times 2]$, $B [2 \times 1]$, $C [1 \times 2]$, $D [1 \times 1]$
 * Hukum Newton:
 $F(t) = M \frac{d^2x(t)}{dt^2} + B \frac{dx(t)}{dt} + Kx(t)$
 $x(t) = x_2 \rightarrow \dot{x}_2 = \dot{x}_1$
 $\frac{dx(t)}{dt} = x_1 \rightarrow \frac{d^2x(t)}{dt^2} = \dot{x}_1$
 $u = M\dot{x}_1 + Bx_1 + Kx_2$
 $\dot{x}_1 = -\frac{B}{M}x_1 - \frac{K}{M}x_2 + \frac{1}{M}u$
 $y = x(t) = x_2$
 $\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{B}{M} & -\frac{K}{M} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{M} \\ 0 \end{bmatrix} u$
 Jadi $A = \begin{bmatrix} -\frac{B}{M} & -\frac{K}{M} \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} \frac{1}{M} \\ 0 \end{bmatrix}$
 $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 0 \end{bmatrix}$

* Contoh Matematika :
 Susunlah Model Ruang Keadaan
 $\dot{x} = Ax + Bu$
 $y = Cx + Du$
 * Dimensi Matrix A, B, C dan D
 * jumlah masukan $m = 2$
 * jumlah keluaran $k = 3$
 * jumlah peubah keadaan $n = 2$
 $A [2 \times 2]$, $B [2 \times 2]$, $C [3 \times 2]$, $D [3 \times 2]$
 Model Ruang Keadaan
 $\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$
 $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$
 $C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

$u \triangleq v_s(t)$
 $y_1 \triangleq v_c(t)$
 $y_2 \triangleq i(t)$
 $x_1 \triangleq i(t)$
 $x_2 \triangleq v_c(t)$

Hukum Ohm

(1) $v_s(t) - v_c(t) = L \frac{di(t)}{dt} + Ri(t)$
 (2) $i(t) = C \frac{dv_c(t)}{dt}$

* Dimensi Matrix

$m = 1$ $k = 2$ $n = 2$
 $A [2 \times 2]$, $B [2 \times 1]$, $C [2 \times 2]$, $D [2 \times 1]$

* Persamaan Keadaan

(1) $u - x_2 = L \dot{x}_1 + R x_1$

$x_1 \triangleq i(t) \rightarrow \frac{di(t)}{dt} = \dot{x}_1$
 $x_2 \triangleq v_c(t) \rightarrow \frac{dv_c(t)}{dt} = \dot{x}_2$

$\dot{x}_1 = -\frac{R}{L}x_1 - \frac{1}{L}x_2 + \frac{1}{L}u$

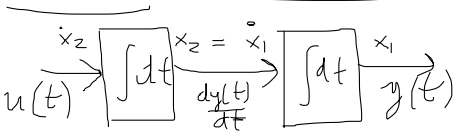
* Persamaan Keluaran

(2) $x_1 = C \dot{x}_2 \rightarrow \dot{x}_2 = \frac{1}{C}x_1$

$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$
 $y_1 = v_c(t) \rightarrow y_1 = x_2$
 $y_2 = i(t) \rightarrow y_2 = x_1$

$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u$
 $A [2 \times 2]$ $B [2 \times 1]$

Contoh: "Double Integrator"



State assignment:

$$x_2 \triangleq y(t) \rightarrow \dot{x}_1 = \frac{dy(t)}{dt}$$

$$x_1 \triangleq \frac{dy(t)}{dt} \rightarrow \dot{x}_1 = x_2$$

$$u \triangleq \ddot{x}_1 \rightarrow \dot{x}_2 = u$$

$$y = y(t) = x_1$$

Perintah MATLAB:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$$

Perintah MATLAB: `ss2tf(A,B,C,D)`
Model Ruang Keadaan dari "Double Integrator"

Model Nisbah Alam!

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{adj[sI - A]}{|sI - A|}$$

$$= \frac{\begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix}}{s^2}$$

$$= \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix}$$

$$G(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0$$

$$= \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0 = \frac{1}{s^2}$$

$$Y(s) = C X(s) + D U(s)$$

$$= C [sI - A]^{-1} B U(s) + D U(s)$$

$$= [C [sI - A]^{-1} B + D] U(s)$$

$$Y(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0$$

$$= \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0 = \frac{1}{s^2}$$

Transfer Function:

$$G(s) = [C [sI - A]^{-1} B + D]$$

khusus SISO

Transf. Laplace:

$$\begin{bmatrix} s X_1(s) \\ s X_2(s) \\ \vdots \\ s X_n(s) \end{bmatrix} = A \begin{bmatrix} X_1(s) \\ X_2(s) \\ \vdots \\ X_n(s) \end{bmatrix} + B U(s)$$

$$\begin{bmatrix} s X_1(s) \\ s X_2(s) \\ \vdots \\ s X_n(s) \end{bmatrix} - A \begin{bmatrix} X_1(s) \\ X_2(s) \\ \vdots \\ X_n(s) \end{bmatrix} = B U(s)$$

$$sI X(s) - A X(s) = B U(s)$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$sI = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix}$$

Sebagai contoh, berikut ini adalah penentuan peubah keadaan yang menghasilkan bentuk matriks A khusus yang disebut "Jordan Companion matrix"

$u \triangleq u(t)$, masukkan $U(s) = \frac{Y(s)}{G(s)}$
 $y \triangleq y(t)$, keluarkan $Y(s) = \frac{Y(s)}{G(s)}$
 $x_1 \triangleq \frac{dx(t)}{dt} = \dot{x}_1 \rightarrow \dot{x}_1 = x_2$
 $x_2 \triangleq \frac{dx(t)}{dt} = \dot{x}_2 \rightarrow \dot{x}_2 = x_3$
 $\vdots$
 $x_{n-2} \triangleq \frac{dx(t)}{dt} = \dot{x}_{n-2} \rightarrow \dot{x}_{n-2} = x_{n-1}$
 $x_{n-1} \triangleq \frac{dx(t)}{dt} = \dot{x}_{n-1} \rightarrow \dot{x}_{n-1} = x_n$
 $x_n \triangleq \frac{dx(t)}{dt} = \dot{x}_n \rightarrow \dot{x}_n = a_n x_1 + a_{n-1} x_2 + \dots + a_1 x_n + a_0 x(t)$
 $u = a_n \frac{dx(t)}{dt} + a_{n-1} x_1 + a_{n-2} x_2 + \dots + a_1 x_n + a_0 x(t)$
 $\dot{x}_n = \frac{1}{a_n} (u - a_{n-1} x_1 - a_{n-2} x_2 - \dots - a_1 x_n - a_0 x(t))$

Persamaan Keadaan:

$\dot{x}_1 = x_2$
 $\dot{x}_2 = x_3$
 $\vdots$
 $\dot{x}_{n-1} = x_n$
 $\dot{x}_n = -\frac{a_0}{a_n} x_1 - \frac{a_1}{a_n} x_2 - \dots - \frac{a_{n-1}}{a_n} x_n + \frac{1}{a_n} u$

$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & -\frac{a_2}{a_n} & \dots & -\frac{a_{n-1}}{a_n} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \frac{1}{a_n} \end{bmatrix} u$

* Pers. Keluaran.
 Dari pers. (1):
 $Y(s) = b_m s^m X(s) + b_{m-1} s^{m-1} X(s) + \dots + b_1 s X(s) + b_0 X(s)$

Transf. Laplace Inverse:
 $y(t) = b_m \frac{d^m x(t)}{dt^m} + b_{m-1} \frac{d^{m-1} x(t)}{dt^{m-1}} + \dots + b_1 \frac{dx(t)}{dt} + b_0 x(t)$

$y = b_m x_1 + b_{m-1} x_2 + \dots + b_1 x_m + b_0 x_{m+1}$
 $y = \begin{bmatrix} b_0 & b_1 & \dots & b_m & b_{m+1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \\ x_{m+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} u$

Jika m=n-1 maka matriks C terisi penuh

Contoh: Tentukan model Ruang Keadaan dengan matriks A berbentuk Matriks Kawanan Jordan (Jordan Companion) dan model Nisbah Alih:

$G(s) = \frac{s^5 + 3s^3 + 5s + 1}{2s^8 + 4s^4 + 6s^2 + 8s + 10}$
 $m=5$ $n=8$ Kasus $m < n$
 $b_0=1$ $a_0=10$
 $b_1=5$ $a_1=0$
 $b_2=0$ $a_2=8$
 $b_3=3$ $a_3=0$
 $b_4=0$ $a_4=6$
 $b_5=1$ $a_5=0$
 $a_6=4$
 $a_7=0$
 $a_8=2$

$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{10}{2} & -\frac{4}{2} & -\frac{6}{2} & -\frac{8}{2} & -\frac{0}{2} & -\frac{0}{2} & -\frac{0}{2} & -\frac{0}{2} \end{bmatrix}$

$C = \begin{bmatrix} 1 & 5 & 0 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
 $D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

* Kasus $m < n$
 * Kasus $m = n$
 * Kasus $m > n$

$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}$
 Jika $Y(s) = (b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0) X(s)$
 maka $U(s) = (a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0) X(s)$

Jadi:
 $(2) \rightarrow U(s) = a_n s^n X(s) + a_{n-1} s^{n-1} X(s) + a_{n-2} s^{n-2} X(s) + \dots + a_1 s X(s) + a_0 X(s)$
 Trans. Laplace Inverse:
 $u(t) = a_n \frac{d^n x(t)}{dt^n} + a_{n-1} \frac{d^{n-1} x(t)}{dt^{n-1}} + a_{n-2} \frac{d^{n-2} x(t)}{dt^{n-2}} + \dots + a_1 \frac{dx(t)}{dt} + a_0 x(t)$

Penentuan peubah keadaan
 state assignment

Catatan: "state assignment" bisa bermacam-macam, karena itu model Ruang Keadaan yang dihasilkan bisa bermacam-macam pula.

* Solusi Persamaan Keadaan

$$\frac{dx(t)}{dt} = \dot{x} = Ax + Bu, \quad x(0)$$

Solusi $x(t) = \dots$

Mulai dengan kasus 'skalar', tanpa masukan u :

$$\frac{dx(t)}{dt} = \dot{x} = a x(t), \quad x(0) = x_0$$

$$\int \frac{dx(t)}{x(t)} = \int a dt = a \int dt = at$$

$$\ln [x(t)] = at + K$$

$$e^{\log [x(t)]} = e^{at + K}$$

$$x(t) = e^{at + K} = e^K \cdot e^{at} = A e^{at}$$

$$t=0 \rightarrow x(0) = A e^0 = A$$

$$x_0 = A$$

$$\text{Solusi: } x(t) = x_0 e^{at}$$

check: $\frac{dx(t)}{dt} = a x_0 e^{at} = a x(t)$
checked

* Kasus mcn
Matriks A dan B tidak haruslah
Matriks $\in [1 \times n] \sim \in [1 \times n]$



Contoh: $G(s) = \frac{s^8 + 3s^3 + 5s + 1}{2s^8 + 4s^6 + 6s^4 + 8s^2 + 10}$

A & B sama dengan contoh sebelumnya

Pembagian

$$\begin{array}{r} 0,5 \\ 2s^8 + 4s^6 + 6s^4 + 8s^2 + 10 \overline{) 5s^8 + 3s^3 + 5s + 1} \\ \underline{2s^8 + 4s^6 + 6s^4 + 8s^2 + 10} \end{array}$$

$$G(s) = \frac{-2s^6 - 3s^3 + 3s^3 - 4s^2 + 5s - 4}{2s^8 + 4s^6 + 6s^4 + 8s^2 + 10} + 0,5$$

$$\frac{8}{3} = \frac{2}{3} + 2$$

Kasus $m < n$

$$C = [-4 \ 5 \ -4 \ 3 \ -3 \ 0 \ -2 \ 0] \quad [1 \times 8]$$

$$D = [0,5] \quad [1 \times 1]$$

Misalnya

$$x(t) = x_0 \left(1 + at + \frac{1}{2} a^2 t^2 + \frac{1}{6} a^3 t^3 + \frac{1}{24} a^4 t^4 + \dots \right)$$

$$\frac{dx(t)}{dt} = x_0 \left(a + a^2 t + \frac{1}{2} a^3 t^2 + \frac{1}{6} a^4 t^3 + \dots \right)$$

$$= a x_0 \left(1 + at + \frac{1}{2} a^2 t^2 + \frac{1}{6} a^3 t^3 + \dots \right)$$

$$= a x(t)$$

maika jelaslah:

$$e^{at} = 1 + at + \frac{1}{2} a^2 t^2 + \frac{1}{6} a^3 t^3 + \dots$$

$$e^1 = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \dots \approx 2,718 \dots$$