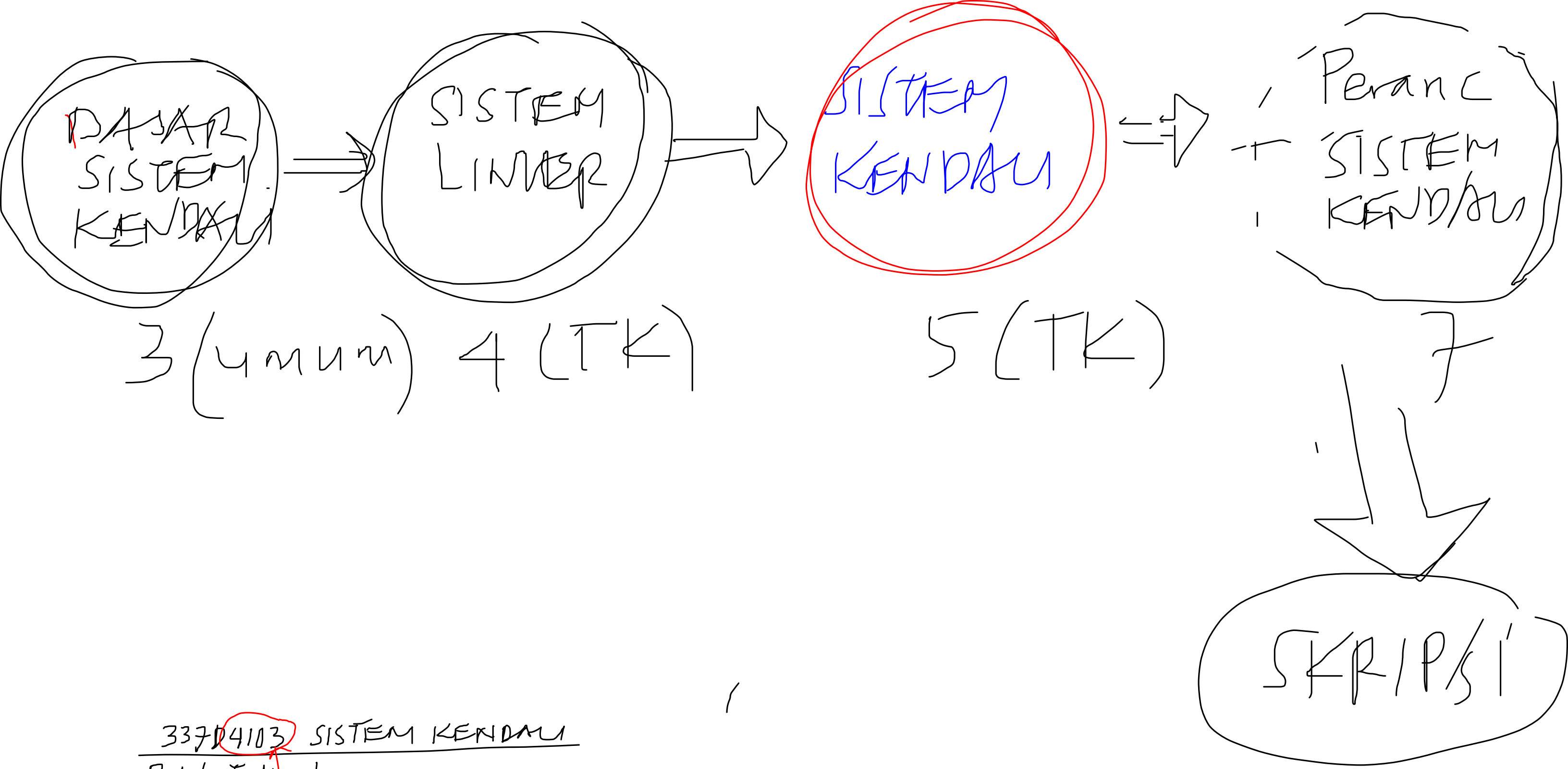


337D4103 SISTEM KENDALI

Rapat Jadwal

3SKS. * 1 SKS : pekan 1 s/d 8 RHZ
1 SKS . pekan 9 s/d 16 AEU
1 SKS . PRAKTIKUM.
- Praktikum Individu
- Praktikum Kelompok

3SKS

Penilaian N.A = $\frac{\text{RHZ} + \text{AEU} + \text{Praktikum}}{3}$ ← kurang lebih

Link : http://www.unhas.ac.id/rhiza/arsip/kuliah/

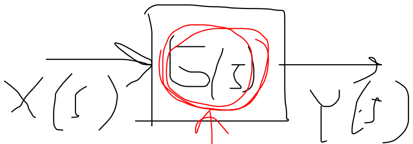
RHZ : TEST 1 (pekan ke-8)
TEST 2 (pekan ke-16)

→ Nilai RHZ = $\frac{\text{Test I} + \text{Test 2}}{2}$

Silabus dapat dilihat di :

http://www.unhas.ac.id/rhiza/arsip/kuliah/silabus-2008/

* Hanya memperlihatkan hubungan input — output saja, tidak memperlihatkan DINAMIKA INTERNAL sistem



$$G(s) = \frac{Y(s)}{X(s)}$$

* Model Ruang Keadaan bersifat lebih UMUM (general) dari Model Nisbah Alih: semua sistem yang dapat dimodelkan dengan model Nisbah Alih akan dapat dimodelkan dengan Model Ruang Keadaan, tapi belum tentu sebaliknya.

* Model Nisbah Alih hanya dapat memodelkan sistem linier. Model Ruang Keadaan dapat memodelkan sistem Tak Linier juga.

LTV

LTI saja

Apa kelebihan Model Nisbah Alih??

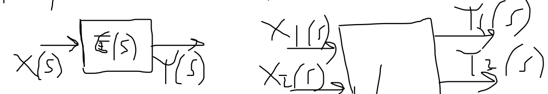
Praktis dan sederhana

Model Ruang Keadaan

State Space Model

Kelemahan Model Nisbah-Alih (Transfer Function):

* Hanya cocok untuk sistem "sederhana", SISO



4 (empat) Nisbah Alih

$$G_1(s) = \frac{Y_1(s)}{X_1(s)}\bigg|_{X_2(s)=0}$$

$$G_2(s) = \frac{Y_2(s)}{X_1(s)}\bigg|_{X_2(s)=0}$$

$$G_3(s) = \frac{Y_1(s)}{X_2(s)}\bigg|_{X_1(s)=0}$$

$$G_4(s) = \frac{Y_2(s)}{X_2(s)}\bigg|_{X_1(s)=0}$$

Model Ruang Keadaan bisa untuk sistem MIMO

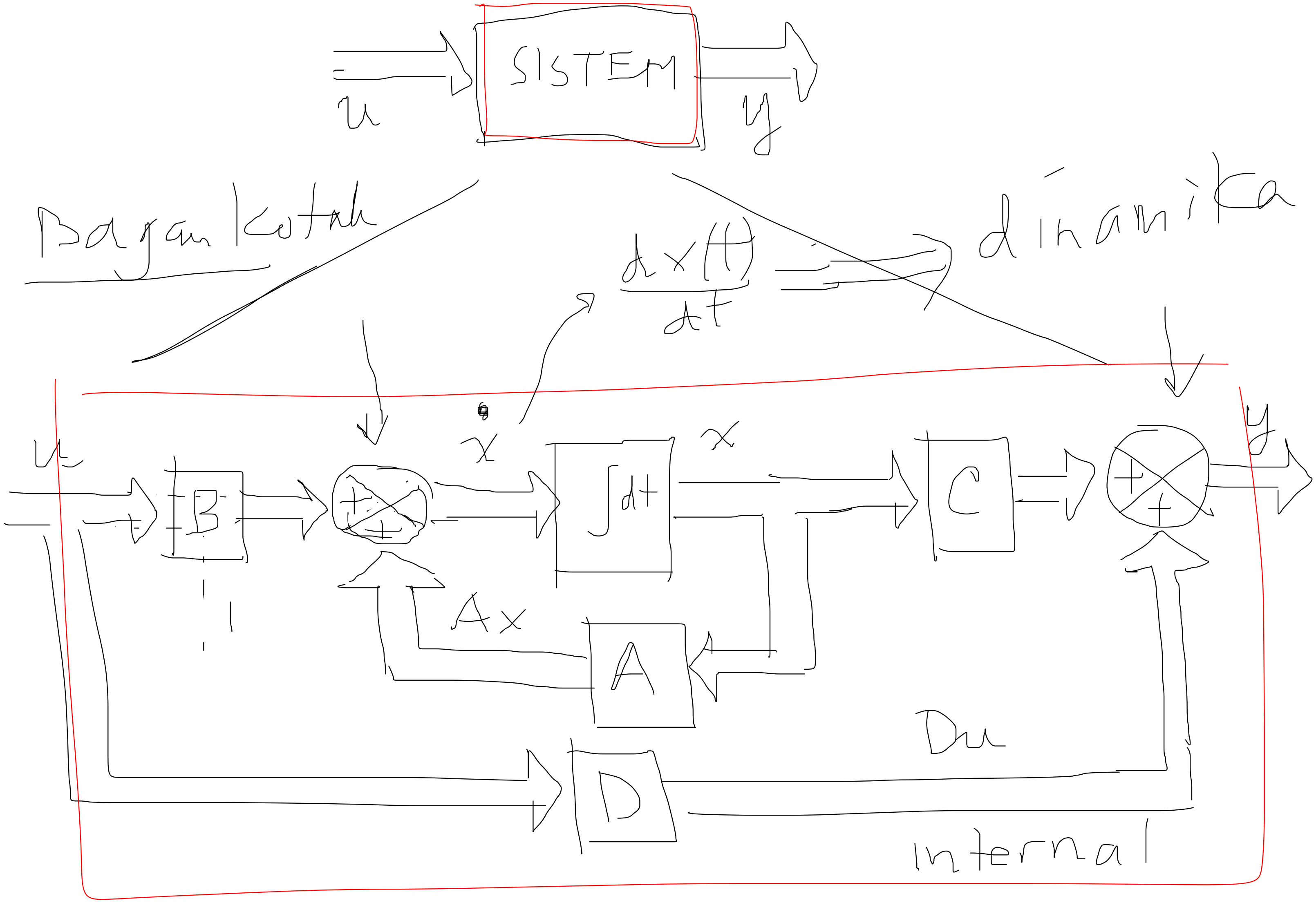
* Analisis dengan model Nisbah Alih lebih efektif untuk "pencil & paper", sulit untuk simulasi dengan komputer

Tabel Laplace

$$\mathcal{L}^{-1}\frac{1}{s+2} = \ldots ? \to e^{-2t}$$

$$\frac{1}{s+a} \quad e^{-at}$$

Model Ruang Keadaan (State space)



Persamaan :   Keadaan

(1) Persamaan (Masukan) : $\dot{x} = Ax + Bu$

(2) Persamaan Keluaran : $y = Cx + Du$

Aljabar Matrix (Aljabar Linier)

$$\underset{[n\times 1]}{\dot{x}} = \underset{[n\times 1]}{Ax} + \underset{[n\times 1]}{Bu}$$

Dimensi: $A[n\times n]$, $B[n\times m]$, $x\ [n\times 1]$, $u\ [m\times 1]$

$$\underset{[k\times 1]}{y} = \underset{[k\times 1]}{Cx} + \underset{[k\times 1]}{Du}$$

$C[k\times n]$, $D[k\times m]$, $x\ [n\times 1]$

Definisi:

$u \triangleq u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}$ vektor kolom $[m\times 1]$

masukan INPUT

$y \triangleq y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_k(t) \end{bmatrix}$ vektor kolom $[k\times 1]$

keluaran OUTPUT

$x \triangleq x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$ vektor kolom $[n\times 1]$

peubah keadaan (state variable)

$\dot{x} \triangleq \dfrac{dx(t)}{dt} = \begin{bmatrix} dx_1(t)/dt \\ dx_2(t)/dt \\ \vdots \\ dx_n(t)/dt \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix}$ vektor kolom $(n\times 1)$

Contoh: Suatu sistem mempunyai 3 masukan, 4 keluaran dan 5 peubah keadaan. Tentukan dimensi matrix A, B, C dan D

Jawab: $m = 3$, $n = 5$, $k = 4$

$A[n\times n] \longrightarrow A[5\times 5]$

$B[n\times m] \longrightarrow B[5\times 3]$

$C[k\times n] \longrightarrow C[4\times 5]$

$D[k\times m] \longrightarrow D[4\times 3]$

Catatan:

* Jika matrix A, B, C dan D semuanya berisi KONSTANTA, maka sistem linier yang "time-invariant" (LTI)

* Jika di antara matrix A, B, C dan D ada yang berisi peubah fungsi waktu t, maka sistem linier "time-varying" (LTV)

* Selain itu sistem tak linier

* Model Nisbah Alih hanya dapat memodelkan sistem LTI saja

Next Contoh

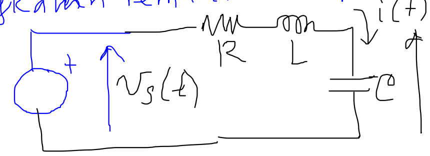
Next

ss2tf !

tf2ss !

* Contoh Elektrik

Rangkaian Pemuatan Kapasitor (RLC seri)



Susunlah Model Ruang Keadaan.

$$\overset{\circ}{x} = Ax + Bu$$
$$y = Cx + Du$$

dengan penunjukan (state assignment)

$u \triangleq v_s(t)$

$y_1 \triangleq v_C(t)$

$y_2 \triangleq i(t)$

$x_1 \triangleq i(t)$

$x_2 \triangleq v_C(t)$

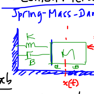
Hukum Ohm.

(1) $v_s(t) - v_C(t) = L\dfrac{di(t)}{dt} + Ri(t)$

(2) $i(t) = C\dfrac{dv_C(t)}{dt}$

* Contoh Mekanik.

Spring-Mass-Damper System



Susunlah Model Ruang Keadaan

dengan: $u \triangleq F(t)$, 1 masukan, $m = 1$

$y \triangleq x(t)$, 1 keluaran $k = 1$

$\begin{bmatrix} \dot{x} = Ax + Bu \\ y = Cx + Du \end{bmatrix}$ $\begin{bmatrix} x_1 \triangleq \frac{dx(t)}{dt} \\ x_2 \triangleq x(t) \end{bmatrix}$, 2 peubah keadaan $n = 2$

Jawab

* Dimensi matrix $A[2\times2]$, $B[2\times1]$, $C[1\times2]$, $D[1\times1]$
* Hukum Newton:

$$F(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t)$$

$x(t) = x_2 \rightarrow \dfrac{dx(t)}{dt} = \dot{x}_2 \rightarrow \dot{x}_2 = x_1$

$\dfrac{dx(t)}{dt} = x_1 \rightarrow \dfrac{d^2x(t)}{dt^2} = \dot{x}_1$

$u = M\dot{x}_1 + Bx_1 + Kx_2$

$\dot{x}_1 = -\dfrac{B}{M}x_1 - \dfrac{K}{M}x_2 + \dfrac{1}{M}u$

$y = x(t) \longrightarrow y = x_2$

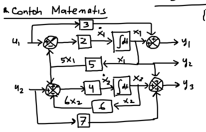
$$\overset{\circ}{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{B}{M} & -\frac{K}{M} \\ 1 & \varnothing \end{bmatrix}}_{A[2\times2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{M} \\ \varnothing \end{bmatrix}}_{B[2\times1]} u$$

$$y = \underbrace{\begin{bmatrix} \varnothing & 1 \end{bmatrix}}_{C[1\times2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \varnothing \end{bmatrix}}_{D[1\times1]} u$$

Jadi: $A = \begin{bmatrix} -\frac{B}{M} & -\frac{K}{M} \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1/M \\ 0 \end{bmatrix}$

$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 0 \end{bmatrix}$

* Contoh Matematis



Susunlah:

Model Ruang Keadaan

$\overset{\circ}{x} = Ax + Bu$

$y = Cx + Du$

a) Dimensi Matrix A, B, C dan D

* jumlah masukan $m = 2$
* jumlah keluaran $k = 3$
* jumlah peubah keadaan $n = 2$

$A[2\times2]$, $B[2\times2]$, $C[3\times2]$, $D[3\times2]$

Model Ruang Keadaan $\begin{bmatrix} \dot{x} = Ax + Bu \\ y = Cx + Du \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

b) Pers Keadaan

$\dot{x}_1 = 2[-5x_1 + u_1] = -10x_1 + 2u_1$

$\dot{x}_2 = 4[+5x_1 - 6x_2 + u_2]$

$= 20x_1 - 24x_2 + 4u_2$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -10 & 0 \\ 20 & -24 \end{bmatrix}}_{A[2\times2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}}_{B[2\times2]} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$A = \begin{bmatrix} -10 & 0 \\ 20 & -24 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$

c) Pers. keluaran

$y_1 = x_1 + 3u_1$

$y_2 = x_1$

$y_3 = x_1 + x_2 + 7u_2$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}}_{C[3\times2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 3 & 0 \\ 0 & 0 \\ 0 & 7 \end{bmatrix}}_{D[3\times2]} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$C = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \\ 0 & 7 \end{bmatrix}$

* Dimensi Matrix.

$m = 1$ $k = 2$ $n = 2$

$A[2\times2]$, $B = [2\times1]$, $C[2\times2]$, $D[2\times1]$

* Persamaan Keadaan

$x_1 \triangleq i(t) \rightarrow \dfrac{di(t)}{dt} = \dot{x}_1$

$x_2 \triangleq v_C(t) \rightarrow \dfrac{dv_C(t)}{dt} = \dot{x}_2$

(1) $u - x_2 = L\dot{x}_1 + Rx_1$

$$\dot{x}_1 = -\frac{R}{L}x_1 - \frac{1}{L}x_2 + \frac{1}{L}u$$

(2) $x_1 = C\dot{x}_2 \longrightarrow \dot{x}_2 = \dfrac{1}{C}x_1$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & \varnothing \end{bmatrix}}_{A[2\times2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{L} \\ \varnothing \end{bmatrix}}_{B[2\times1]} u$$

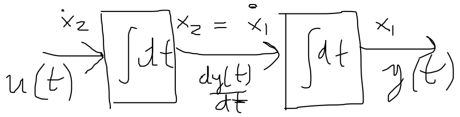
* Persamaan Keluaran

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \varnothing & 1 \\ 1 & \varnothing \end{bmatrix}}_{C[2\times2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \varnothing \\ \varnothing \end{bmatrix}}_{D[2\times1]} u$$

$y_1 = v_C(t) \rightarrow y_1 = x_2$

$y_2 = i(t) \rightarrow y_2 = x_1$

Contoh: "Double Integrator"



State assignment:

$x_1 \triangleq y(t) \rightarrow \dot{x}_1 = \frac{dy(t)}{dt}$

$x_2 \triangleq \frac{dy(t)}{dt} \qquad \dot{x}_1 = x_2$

$u \triangleq \dot{x}_2 \qquad \dot{x}_2 = u$

$y = y(t) = x_1$

Pers keadaan

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Pers. keluaran: $\quad A[2\times2] \quad B[2\times1]$

$$y = \begin{bmatrix} 1 & \emptyset \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [\emptyset]\, u$$

$C[1\times2] \qquad D[1\times1]$

ss2tf(A,B,C,D)

Model Ruang Keadaan dari "Double Integrator"

>>> help ss2tf
>>> help tf2ss } MATLAB functions Control Tool Box

"State Space" to "Transfer Function"
"Transfer Function" to "State Space"

$X(s) \rightarrow \boxed{G(s)} \rightarrow Y(s) \quad \Longleftrightarrow \quad u \rightarrow \boxed{\begin{matrix}\dot{x} = Ax + Bu \\ y = Cx + Du\end{matrix}} \rightarrow y$

Model Nisbah Alih (Transfer Function) — Model Ruang Keadaan (State Space)

Semua Model Nisbah Alih dapat dibuat Model Ruang Keadaan, tapi tidak sebaliknya.

* ss2tf — Model Ruang Keadaan yang dapat diubah menjadi model Nisbah Alih hanya model sistem SISO (single input single output)

$\Rightarrow A[n\times n],\ B[n\times1],\ C[1\times n],\ D[1\times1]$

$m = 1$ dan $k = 1$ (SISO)

$\dot{x} = Ax + Bu \qquad u \triangleq u(t)$ single input

$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \vdots \\ \frac{dx_n(t)}{dt} \end{bmatrix} = \begin{bmatrix} & & \\ & A[n\times n] & \\ & & \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{B[n\times1]} u(t)$$

Transf. Laplace:

$$\begin{bmatrix} sX_1(s) \\ sX_2(s) \\ \vdots \\ sX_n(s) \end{bmatrix} = A \begin{bmatrix} X_1(s) \\ X_2(s) \\ \vdots \\ X_3(s) \end{bmatrix} + BU(s)$$

$$\underbrace{\begin{bmatrix} sX_1(s) \\ sX_2(s) \\ \vdots \\ sX_n(s) \end{bmatrix}}_{[n\times1]} - \underset{[n\times n]}{A} \underbrace{\begin{bmatrix} X_1(s) \\ X_2(s) \\ \vdots \\ X_n(s) \end{bmatrix}}_{[n\times1]} = \underset{[n\times1]}{B}\ \underset{[1\times1]}{U(s)}$$

$sIX(s) - AX(s) = BU(s)$

$$I = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & \ddots\ 1 \end{bmatrix}$$

$$sI = \underbrace{\begin{bmatrix} s & & 0 \\ & s & \\ 0 & & \ddots\ s \end{bmatrix}}_{[n\times n]}$$

Model Nisbah Alih:

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\operatorname{adj}[sI - A]}{|I - A|} = \frac{\begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix}}{s^2} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$sIX(s) - AX(s) = BU(s)$

$[sI - A]X(s) = BU(s)$

$\underbrace{[sI-A]^{-1}[sI-A]}_{I}X(s) = [sI-A]^{-1}BU(s)$

$X(s) = [sI-A]^{-1}BU(s)$

$y = Cx + Du$

Transf. Laplace:

$Y(s) = CX(s) + DU(s)$

$= C[sI - A]^{-1}BU(s) + DU(s)$

$= \{C[sI - A]^{-1}B + D\}U(s)$

Transfer Function:

$$\boxed{G(s) = [C[sI - A]^{-1}B + D]}$$

Khusus SISO

$$G(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0$$

$$= \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0 = \frac{1}{s^2}$$

Sebagai contoh, berikut ini adalah penentuan peubah keadaan yang menghasilkan bentuk matrix A khusus yang disebut "Jordan Companion matrix"

$u \triangleq u(t)$, masukan $U(s) = \mathcal{L}[u(t)]$ $G(s) = \frac{Y(s)}{U(s)}$

$y \triangleq y(t)$, keluaran $Y(s) = \mathcal{L}[y(t)]$

$x_1 \triangleq x(t)$

$x_2 \triangleq \frac{dx(t)}{dt} = \dot{x}_1 \longrightarrow \dot{x}_1 = x_2$

$x_3 \triangleq \frac{d^2x(t)}{dt^2} = \dot{x}_2 \longrightarrow \dot{x}_2 = x_3$

$\vdots$

$x_{n-2} \triangleq \frac{d^{n-3}x(t)}{dt^{n-3}} = \dot{x}_{n-3} \longrightarrow \dot{x}_{n-3} = x_{n-2}$

$x_{n-1} \triangleq \frac{d^{n-2}x(t)}{dt^{n-2}} = \dot{x}_{n-2} \longrightarrow \dot{x}_{n-2} = x_{n-1}$

$x_n \triangleq \frac{d^{n-1}x(t)}{dt^{n-1}} = \dot{x}_{n-1} \longrightarrow \dot{x}_{n-1} = x_n$

$$u(t) = a_n \frac{d^n x(t)}{dt^n} + a_{n-1}\frac{d^{n-1}x(t)}{dt^{n-1}} + a_{n-2}\frac{d^{n-2}x(t)}{dt^{n-2}} + \dots + a_1\frac{dx(t)}{dt} + a_0 x(t)$$

$$u = a_n \frac{d^n x(t)}{dt^n} + a_{n-1}x_n + a_{n-2}x_{n-1} + \dots + a_1 x_2 + a_0 x_1$$

$$\dot{x}_n = \frac{d^n x(t)}{dt^n} = -\frac{1}{a_n}\left(a_0 x_1 + a_1 x_2 + a_2 x_3 + \dots + a_{n-2}x_{n-1} + a_{n-1}x_n\right) + \frac{1}{a_n}u$$

Persamaan Keadaan:

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\vdots$

$\dot{x}_{n-1} = x_n$

$$\dot{x}_n = -\frac{a_0}{a_n}x_1 - \frac{a_1}{a_n}x_2 - \frac{a_2}{a_n}x_3 - \dots - \frac{a_{n-1}}{a_n}x_n + \frac{1}{a_n}u$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ & & & \ddots & \\ 0 & 0 & 0 & \dots & 1 \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & -\frac{a_2}{a_n} & \dots & -\frac{a_{n-1}}{a_n} \end{bmatrix}}_{A[n\times n]} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ \frac{1}{a_n} \end{bmatrix}}_{B[n\times 1]} u$$

* Pers. Keluaran.

Dari pers. (1):

$$Y(s) = b_m s^m X(s) + b_{m-1}s^{m-1}X(s) + \dots + b_1 s X(s) + b_0 X(s)$$

Transf. Laplace Inverse.

$$y(t) = b_m \frac{d^m x(t)}{dt^m} + b_{m-1}\frac{d^{m-1}x(t)}{dt^{m-1}} + \dots + b_1\frac{dx(t)}{dt} + b_0 x(t)$$

$$y = b_0 x_1 + b_1 x_2 + \dots + b_{m-1}x_m + b_m x_{m+1}$$

$$y = \underbrace{[b_0\, b_1\, b_2 \dots b_{m-1}\, b_m\, 0 \dots 0]}_{C[1\times n]} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \\ x_{m+1} \\ \vdots \\ x_n \end{bmatrix} + \underbrace{[\varnothing]}_{D[1\times 1]} u$$

Jika $m = n-1$ maka matrix C terisi penuh

*tf2ss Semua Model Nisbah Alih dapat diubah menjadi model Ruang Keadaan

$U(s) \rightarrow \boxed{G(s)} \rightarrow Y(s) \Rightarrow u \rightarrow \boxed{\begin{matrix}\dot{x} = Ax + Bu \\ y = Cx + Du\end{matrix}} \rightarrow y$

SISO

Bentuk Umum Model Nisbah Alih

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + b_{m-2}s^{m-2} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_1 s + a_0}$$

m, n : bilangan bulat, $\boxed{m \le n}$ tidak ada $m > n$

$a_0, a_1, a_2, \dots, a_{n-1}, a_n$ koefisien REAL

$b_0, b_1, b_2, \dots, b_{m-1}, b_m$ Koefisien REAL

* Kasus $m < n$
* Kasus $m = n$

$X(s) = \mathcal{L}\, x(t)$

* Kasus $m < n$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{(b_m s^m + b_{m-1}s^{m-1} + b_{m-2}s^{m-2} + \dots + b_1 s + b_0)X(s)}{(a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_1 s + a_0)X(s)}$$

Jika $Y(s) = (b_m s^m + b_{m-1}s^{m-1} + b_{m-2}s^{m-2} + \dots + b_1 s + b_0)X(s)$ → (1)

maka $U(s) = (a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_1 s + a_0)X(s)$ → (2)

sebab : $\left[\frac{k}{m} = \frac{1}{2}\right] \longrightarrow$ [jika $k = 1$, maka $m = 2$]

Jadi

$$(2) \longrightarrow U(s) = a_n s^n X(s) + a_{n-1}s^{n-1}X(s) + a_{n-2}s^{n-2}X(s) + \dots + a_1 s X(s) + a_0 X(s)$$

Trans. Laplace Inverse.

$$u(t) = a_n \frac{d^n x(t)}{dt^n} + a_{n-1}\frac{d^{n-1}x(t)}{dt^{n-1}} + a_{n-2}\frac{d^{n-2}x(t)}{dt^{n-2}} + \dots + a_1\frac{dx(t)}{dt} + a_0 x(t)$$

Penentuan peubah keadaan

state assignment

Contoh : Tentukan model Ruang Keadaan dengan matrix A berbentuk Matrix Kawanan Jordan (Jordan Companion) dan model Nisbah Alih :

$$G(s) = \frac{s^5 + 3s^3 + 5s + 1}{2s^8 + 4s^6 + 6s^4 + 8s^2 + 10}$$

$m = 5$ $n = 8$ Kasus $m < n$

$b_0 = 1$, $b_1 = 5$, $b_2 = 0$, $b_3 = 3$, $b_4 = 0$, $b_5 = 1$

$a_0 = 10$, $a_1 = 0$, $a_2 = 8$, $a_3 = 0$, $a_4 = 6$, $a_5 = 0$, $a_6 = 4$, $a_7 = 0$, $a_8 = 2$

$$\begin{bmatrix} \dot{x} = Ax + Bu \\ y = Cx + Du \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_8 \end{bmatrix}$$

$$\underset{[8\times 8]}{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{10}{2} & 0 & -\frac{8}{2} & 0 & -\frac{6}{2} & 0 & -\frac{4}{2} & 0 \end{bmatrix}$$

$$\underset{[8\times 1]}{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

$\underset{[1\times 8]}{C} = [1\ 5\ 0\ 3\ 0\ 1\ 0\ 0]$

$\underset{[1\times 1]}{D} = [\varnothing]$

Catatan : "state assignment" bisa bermacam², karena itu model Ruang Keadaan yang dihasilkan bisa ber-macam² pula.

*Solusi Persamaan Keadaan

$$\boxed{\frac{dx(t)}{dt} = \dot{x} = Ax + Bu}\ ,\ x(0)$$

solusi $x(t) = \ldots\ ?$

Mulai dengan kasus "skalar", tanpa masukan u:

$$\frac{dx(t)}{dt} = \dot{x} = ax(t)\ ,\ x(0) = x_0$$

$$\int \frac{dx(t)}{x(t)} = \int a\,dt = a\int dt = at$$

$$\ln[x(t)] = at + K$$

$$^{e}\log[x(t)] = at + K$$

$$x(t) = e^{(at+K)}$$

$$x(t) = e^{K} \cdot e^{at}$$

$$= Ae^{at}$$

$$t = 0 \rightarrow x(0) = Ae^{0} = A$$

$$x_0 = A$$

Solusi: $\boxed{x(t) = x_0 e^{at}}$

check: $\frac{dx(t)}{dt} = a x_0 e^{at} = a x(t)$

checked

$$\int \frac{dx}{x} = \ln x$$

$$\int x\,dx = \frac{1}{2}x^2$$

$$\int dx = x$$

$$\int x^2 dx = \frac{1}{3}x^3$$

$$\int x^3 dx = \frac{1}{4}x^4$$

$$\int x^{-2} dx = -x^{-1} = -\frac{1}{x}$$

$$\int x^{-1} dx = \int \frac{dx}{x} = \ln x$$

$$\int x^{-3} dx = -\frac{1}{2}x^{-2}$$

$$^{a}\log b = c$$

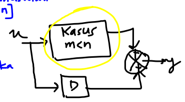
$$b = a^{c}$$

*Kasus m=n

Matrix A dan B tidak masalah

Matrix $C\,[1\times m] \rightarrow C\,[1\times n]$

b_0, b_1, b_2, …, b_{m-1}, b_m } (m+1) angka



Contoh: $G(s) = \frac{s^8 + 3s^3 + 5s + 1}{2s^8 + 4s^6 + 6s^4 + 8s^2 + 10}$ → A & B sama dengan contoh sebelumnya,

Pembagian:

$$2s^8+4s^6+6s^4+8s^2+10 \ \overline{\big)\ s^8 + 3s^3 + 5s + 1}\ \ 0{,}5$$

$$\underline{s^8 + 2s^6 + 3s^4 + 4s^2 + 5}\ -$$

$$-2s^6 - 3s^4 + 3s^3 - 4s^2 + 5s - 4$$

$$G(s) = \frac{-2s^6 - 3s^4 + 3s^3 - 4s^2 + 5s - 4}{2s^8 + 4s^6 + 6s^4 + 8s^2 + 10} + 0{,}5$$

Kasus m<n

$$\frac{8}{3} = \frac{2}{3} + 2$$

$$\underset{[1\times 8]}{C} = [-4\ \ 5\ \ -4\ \ 3\ \ -3\ \ 0\ \ -2\ \ 0]$$

$$\underset{[1\times 1]}{D} = [0{,}5]$$

Misalnya:

$$\boxed{x(t) = x_0\left(1 + at + \frac{1}{2}a^2t^2 + \frac{1}{6}a^3t^3 + \frac{1}{24}a^4t^4\right)}$$

$$\frac{dx(t)}{dt} = x_0\left(a + a^2t + \frac{1}{2}a^3t^2 + \frac{1}{6}a^4t^3 + \ldots\right)$$

$$= a x_0\left(1 + at + \frac{1}{2}a^2t^2 + \frac{1}{6}a^3t^3 + \ldots\right)$$

$$= a\,x(t)$$

maka jelaslah:

$$\boxed{e^{at} = 1 + at + \frac{1}{2}a^2t^2 + \frac{1}{6}a^3t^3 + \ldots}$$

$$e^{1} = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \ldots \approx 2{,}718\ldots$$

*** Kasus MATRIKS**

$\dot{x} = Ax \qquad x = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \quad A\,[n \times n] \qquad x_0 = \begin{bmatrix} x_1(0) \\ x_2(0) \\ \vdots \\ x_n(0) \end{bmatrix}$

$$x(t) = [I + At + \tfrac{1}{2}A^2t^2 + \tfrac{1}{6}A^3t^3 + \tfrac{1}{24}A^4t^4 + \quad)\, x_0$$

$$\frac{dx(t)}{dt} = [A + A^2t + \tfrac{1}{2}A^3t^2 + \tfrac{1}{6}A^4t^3 + \cdots)\, x_0$$

$$= A[I + At + \tfrac{1}{2}A^2t^2 + \tfrac{1}{6}A^3t^3 + \cdots)\, x_0$$

$$\dot{x} = Ax(t)$$

$$\boxed{I + At + \tfrac{1}{2}A^2t^2 + \tfrac{1}{6}A^3t^3 + \cdots \equiv e^{At}} \rightarrow [n \times n]$$

Solusi $\dot{x} = Ax \rightarrow x(t) = e^{At}x_0 \longrightarrow$ solusi khusus

Bagaimana solusi dari $\dot{x} = Ax + Bu$

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$

Misalnya $x(t) = e^{At}c(t)$

maka $\dfrac{dx(t)}{dt} = \underbrace{Ae^{At}c(t)} + e^{At}\dfrac{dc(t)}{dt} = Ax(t) + Bu(t)$

$$\cancel{Ax(t)} + e^{At}\frac{dc(t)}{dt} = \cancel{Ax(t)} + Bu(t)$$

$$e^{At}\frac{dc(t)}{dt} = Bu(t)$$

$$\underbrace{[e^{At}]^{-1}[e^{At}]}_{I} \cdot \frac{dc(t)}{dt} = [e^{At}]^{-1}Bu(t)$$

$$\frac{dc(t)}{dt} = [e^{At}]^{-1}Bu(t)$$

$$\int dc(t) = \int [e^{At}]^{-1}Bu(t)\,dt$$

$$c(t) = \int [e^{At}]^{-1}Bu(t)\,dt$$

$$x(t) = e^{At}c(t)$$

$$= e^{At}\int [e^{At}]^{-1}Bu(t)\,dt \longrightarrow \text{solusi umum}$$

Karena kerumitan SOLUSI ANALITIK dari model Ruang Keadaan maka orang lebih suka mencari SOLUSI NUMERIK menggunakan SIMULASI

SOLUSI:

$$\boxed{x(t) = e^{At}x_0 + e^{At}\int [e^{At}]^{-1}Bu(t)\,dt}$$

di-substitusi ke persamaan keluaran

$$y = Cx + Du$$

Keluaran:

$$\boxed{y(t) = Cx(t) + Du(t)}$$

next! Transformasi SIMILARITAS

Contoh

Suatu sistem dimodelkan dengan model Ruang Keadaan

$$\dot{x} = Ax + Bu \qquad A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$y = Cx + Du \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = [0]$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

dengan matrix $T = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ ubahlah model sehingga peubah keadaannya $\bar{x} = Tx$

$$\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = Tx = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_1 + 2x_2 \end{bmatrix}$$

Matrix T invertible $\det(T) = 2 \neq 0$

$$T^{-1} = \frac{adj(T)}{|T|} = \frac{\begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}}{2} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

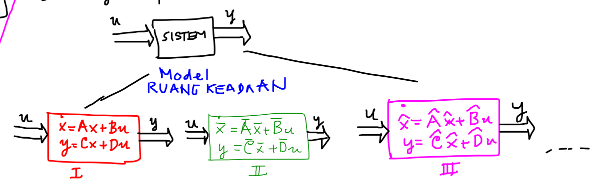
$$\bar{A} = TAT^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 3/2 \end{bmatrix}$$

$$\bar{B} = TB = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\bar{C} = CT^{-1} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad \bar{D} = D = [0]$$

*** Transformasi Similaritas**

Similarity Transformation



Dan sistem yang sama dapat disusun banyak model Ruang Keadaan yang berbeda-beda, tapi sesungguhnya sama (SIMILAR), tergantung pada pemilihan peubah keadaan, $x, \bar{x}, \hat{x}$, dst

Transformasi dari model I ke model II, ke model III dst dengan mengubah peubah keadaan $x \to \bar{x} \to \hat{x} \to \ldots$ disebut Transformasi Similaritas.

Misalnya, Model $\dot{x} = Ax + Bu$, $y = Cx + Du$ akan diubah menjadi

Model $\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u$

$y = \bar{C}\bar{x} + \bar{D}u$

dengan $\bar{x} = Tx$, T suatu matrix yang invertible dengan inverse T^{-1}

$a^{-1} \cdot a = 1$

$\frac{1}{5} \cdot 5 = 1$

$A^{-1} \cdot A = I$

Note! Matrix yang punya inverse:

- * Determinan-nya $\neq 0$
- * Baris/kolom bebas linier
- * Non-singular.

$$T\dot{x} = \bar{A}Tx + \bar{B}u$$

$$y = \bar{C}Tx + \bar{D}u$$

$$T^{-1}T\dot{x} = T^{-1}\bar{A}Tx + T^{-1}\bar{B}u$$

$$\dot{x} = T^{-1}\bar{A}Tx + T^{-1}\bar{B}u = Ax + Bu \longrightarrow \boxed{\begin{aligned} T^{-1}\bar{A}T &= A \\ T^{-1}\bar{B} &= B \end{aligned}}$$

$$y = \bar{C}Tx + \bar{D}u = Cx + Du$$

$$T^{-1}\bar{A}T = A$$

$$TT^{-1}\bar{A}TT^{-1} = TAT^{-1}$$

$$\boxed{\bar{A} = TAT^{-1}}$$

$$T^{-1}\bar{B} = B$$

$$TT^{-1}\bar{B} = TB$$

$$\boxed{\bar{B} = TB}$$

$$\bar{C}T = C \rightarrow \bar{C}TT^{-1} = CT^{-1}$$

$$\boxed{\bar{C} = CT^{-1}}$$

$$\boxed{\begin{aligned} \bar{C}T &= C \\ \bar{D} &= D \end{aligned}}$$