* Bagaimana solusi

$\dot{x} = Ax + Bu$ , $x(0) = x_0$ $\quad u = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}$

$\dot{x} = Ax(t) + Bu(t)$ $\quad A\,[n \times n]$, $B = [n \times m]$

Misalkan $x(t) = e^{At} c(t)$ $\qquad c(t) = \begin{bmatrix} c_1(t) \\ c_2(t) \\ \vdots \\ c_n(t) \end{bmatrix}$

$$\dot{x} = \frac{dx(t)}{dt} = \frac{d[e^{At}]}{dt} c(t) + [e^{At}] \frac{dc(t)}{dt}$$

$$\dot{x} = A \underbrace{e^{At} c(t)} + [e^{At}] \frac{dc(t)}{dt}$$

$$\dot{x} = A x(t) + [e^{At}] \frac{dc(t)}{dt}$$

$$\dot{x} = A x(t) + B u(t) \quad -$$

$$0 = [e^{At}] \frac{dc(t)}{dt} - Bu(t)$$

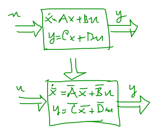
$$[e^{At}] \frac{dc(t)}{dt} = Bu(t)$$

$$\underbrace{[e^{At}]^{-1} [e^{At}]}_{I} \frac{dc(t)}{dt} = [e^{At}]^{-1} Bu(t)$$

$$\frac{dc(t)}{dt} = e^{-At} Bu(t)$$

$$c(t) = \int \underbrace{\underset{[n \times n]}{e^{-At}} \underset{[n \times m]}{B} \underset{[m \times 1]}{u(t)}}_{[n \times 1]} dt$$

* Transformasi SIMILARITAS




$\bar{x} = Tx$

T : matrix non singular
invertible
$\det(T) \neq 0$

Jika matrix A mempunyai nilai-eigen $\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix}$
maka matrix $\bar{A} = TAT^{-1}$ juga
mempunyai nilai eigen yang sama

Oleh karena itu, matrix A akan "similar" dengan matrix diagonal

$$A_d = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

karena ada matrix T sehingga

$$A = TA_dT^{-1}$$

Dari solusi pers diferensial :

$$y(t) = C\left[\bar{x}_0 e^{A_d t} + \int e^{-At} Bu(t)\,dt\right] + Du(t)$$

untuk $u(t) = 0$ (tanpa masukan)

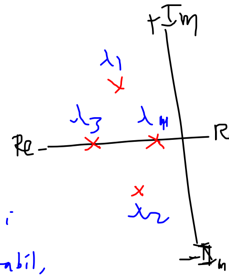
$$y(t) = C\bar{x}_0 e^{A_d t} = C\bar{x}_0 \left[I + A_d t + \tfrac{1}{2} A_d^2 t^2 + \cdots\right]$$

$$= C\bar{x}_0 \begin{bmatrix} 1+\lambda_1 t + \frac{1}{2}\lambda_1^2 t^2 + \cdots & 0 & 0 & \cdots & 0 \\ 0 & 1+\lambda_2 t + \frac{1}{2}\lambda_2^2 t^2 + \cdots & 0 & \cdots & 0 \\ 0 & 0 & 1+\lambda_3 t + \frac{1}{2}\lambda_3^2 t^2 + \cdots & & \\ & & & \ddots & \end{bmatrix}$$

$$= C\bar{x}_0 \begin{bmatrix} e^{\lambda_1 t} & & & & 0 \\ & e^{\lambda_2 t} & & & \\ & & e^{\lambda_3 t} & & \\ 0 & & & \ddots & \\ & & & & e^{\lambda_n t} \end{bmatrix}$$

Jika $\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_n$ semuanya mempunyai bagian REAL yang negatif,
$e^{\lambda_1 t} = e^{(a+jb)t}$, $a < 0$
maka $e^{\lambda_1 t}$ menuju nol untuk $t \to \infty$,
artinya sistem STABIL

Jika ada salah satu saja dari nilai eigen matrix A bagian real-nya non-negatif, maka sistem tidak stabil, karena nilai eigen tersebut akan membuat $e^{\lambda t}$ tidak menuju nol untuk $t \to \infty$




solusi $\dot{x} = Ax + Bu$ adalah solusi $\dot{x} = Ax$ [umum] ditambah solusi $\dot{x} = Ax + Bu$

$$x(t) = x_0 e^{At} + \int e^{-At} Bu(t)\,dt$$

$$y = Cx + Du$$

Keluaran :

$$y(t) = C\left[x_0 e^{At} + \int e^{-At} Bu(t)\,dt\right] + Du(t)$$

Contoh

* Stabilkah

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

* Jawab

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow |\lambda I - A| = 0$$

$$\left|\begin{bmatrix} \lambda & -1 \\ 0 & \lambda \end{bmatrix}\right| = 0$$

$$\lambda^2 = 0 \rightarrow \lambda_1 = \lambda_2 = 0$$

(tidak stabil)

* stabilkah sistem suspensi tanpa shockbreaker $(B = 0)$ ?

$$A = \begin{bmatrix} 0 & -\frac{K}{M} \\ 1 & 0 \end{bmatrix} \qquad [\lambda I - A] = \begin{bmatrix} \lambda & \frac{K}{M} \\ -1 & \lambda \end{bmatrix}$$

$$\det[\lambda I - A] = \lambda^2 + \frac{K}{M} = 0$$

$$\lambda_{1,2} = \pm j\sqrt{\frac{K}{M}}, \text{ non-negatif}$$

TIDAK STABIL

Bagaimana jika tanpa pegas $(K = 0)$

* Stabilkan sistem pemuatan C tanpa R !

$$A = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \qquad [\lambda I - A] = \begin{bmatrix} \lambda & \frac{1}{L} \\ -\frac{1}{C} & \lambda \end{bmatrix}$$

$$\det[\lambda I - A] = 0$$

$$\lambda^2 + \frac{1}{LC} = 0 \rightarrow \lambda^2 = -\frac{1}{LC}$$

$$\lambda_{1,2} = \pm j\sqrt{\frac{1}{LC}}, \text{ non-negatif}$$

TIDAK STABIL

Bagaimana jika tanpa induktor $(L = 0)$