Pers. Keluaran: $y = Cx + Du$

Transf. Laplace: $Y(s) = CX(s) + DU(s)$

$= C[sI - A]^{-1}BU(s) + DU(s)$

$= \underbrace{[C[sI-A]^{-1}B + D]}_{[1\times1]}U(s)$

$\underbrace{[1\times n]\ [n\times n]\ [n\times1]}_{[1\times1]}$

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = [C[sI-A]^{-1}B + D]}$$

$m = 1,\ n = 2,\ k = 1$

$A[2\times2],\ B[2\times1],\ C[1\times2],\ D[1\times1]$

(1) $\dfrac{di(t)}{dt} = -\dfrac{R}{L}i(t) - \dfrac{1}{L}v_0(t) + \dfrac{1}{L}v_i(t)$

$\dot{x}_1 = -\dfrac{R}{L}x_1 - \dfrac{1}{L}x_2 + \dfrac{1}{L}u$

(2) $\dfrac{dv_0(t)}{dt} = \dfrac{1}{C}i(t)$

$\dot{x}_2 = \dfrac{1}{C}x_1$

Pers. keadaan: $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-\frac{R}{L} & -\frac{1}{L}\\ \frac{1}{C} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\frac{1}{L}\\ 0\end{bmatrix}u$

Pers. keluaran: $y = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + [0]u$

Hubungan antara Model Nisbah Alih dgn Model Ruang Keadaan

* Model Ruang Keadaan bersifat lebih umum (general) dari Model Nisbah Alih
* Semua model Nisbah Alih dapat diubah menjadi model Ruang Keadaan, tapi belum tentu sebaliknya
* Model Ruang Keadaan yang dapat diubah menjadi model Nisbah Alih, hanyalah model dari sistem SISO, Linear, Time Invariant

* Model Ruang Keadaan $\longrightarrow$ Nisbah Alih

Perintah: >> ss2tf $\quad U(s) \rightarrow \boxed{G(s)} \rightarrow Y(s)$

Model Ruang Keadaan:

$\begin{cases}\dot{x} = Ax + Bu\\ y = Cx + Du\end{cases} \Rightarrow G(s) = \dfrac{Y(s)}{U(s)}$

$m = 1$, $n \geq 1$, $k = 1$

$u = u(t) = \mathcal{L}^{-1}U(s)$

$y = y(t) = \mathcal{L}^{-1}Y(s)$

$A[n\times n],\ B[n\times1],\ C[1\times n],\ D[1\times1]$

Pers. Keadaan: $\dot{x} = Ax + Bu$

$\dfrac{dx(t)}{dt} = Ax(t) + Bu(t)$

Transf. Laplace: $sX(s) = AX(s) + BU(s)$

$\underbrace{sX(s) - AX(s)}$ $= BU(s)$

$sI = \begin{bmatrix}s & & 0\\ & s & \\ 0 & & s\end{bmatrix}$ $[n\times n]$

$s\begin{bmatrix}1 & & 0\\ & 1 & \\ 0 & & 1\end{bmatrix}$

$[sI - A]X(s) = BU(s)$

$\underbrace{[sI-A]^{-1}[sI-A]}_{I}X(s) = [sI-A]^{-1}BU(s)$

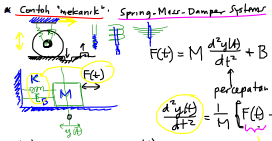
$X(s) = [sI-A]^{-1}BU(s)$

Contoh²

* Contoh "matematis": Tentukan dimensi matrix A, B, C dan D dari sebuah sistem yang punya 3 masukan, 4 peubah keadaan dan 5 keluaran

Jawab: $m = 3,\ n = 4,\ k = 5 \rightarrow A[4\times4]\ \ C[5\times4]\ \ B[4\times3]\ \ D[5\times3]$

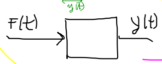
* Contoh "mekanik": Spring-Mass-Damper Systems

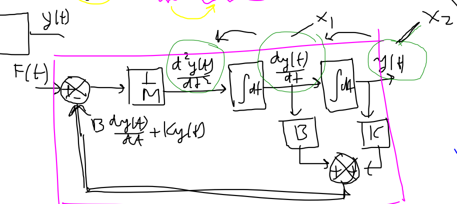


$F(t) = M\dfrac{d^2y(t)}{dt^2} + B\dfrac{dy(t)}{dt} + Ky(t)$

percepatan, kecepatan, posisi

$\dfrac{d^2y(t)}{dt^2} = \dfrac{1}{M}\left\{F(t) - \left[B\dfrac{dy(t)}{dt} + Ky(t)\right]\right\}$





Ditanyakan: Model RUANG KEADAAN sistem di atas!

* SISO $\longrightarrow$ $m = 1$, $k = 1$, $n = 2$

$u \triangleq F(t)$

$y \triangleq y(t)$

$A[2\times2]\quad C[1\times2]$

$B[2\times1]\quad D[1\times1]$

$x = \begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}\frac{dy(t)}{dt}\\ y(t)\end{bmatrix}$ $\rightarrow$ kecepatan, posisi

2 peubah keadaan

Model Ruang Keadaannya:

$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \dot{x} = \underbrace{\begin{bmatrix}-\frac{B}{M} & -\frac{K}{M}\\ 1 & 0\end{bmatrix}}_{A[2\times2]}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}\frac{1}{M}\\ 0\end{bmatrix}}_{B[2\times1]}u$

$y = \underbrace{\begin{bmatrix}0 & 1\end{bmatrix}}_{C[1\times2]}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{[0]}_{D[1\times1]}u$

$\dfrac{d^2y(t)}{dt^2} = \dot{x}_1 = \dfrac{1}{M}\left[F(t) - \left[B\dfrac{dy(t)}{dt} + Ky(t)\right]\right]$

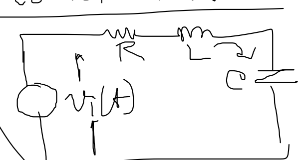
$= \dfrac{1}{M}[u - Bx_1 - Kx_2]$

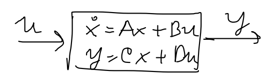
$= -\dfrac{B}{M}x_1 - \dfrac{K}{M}x_2 + \dfrac{1}{M}u$

$\dfrac{dy(t)}{dt} = \dot{x}_2 = x_1$

* Contoh Elektrik:

Pemnatan Kapasitor melalui rangkaian RLC serie





Masukan: $u \triangleq v_i(t)$

Keluaran: $y \triangleq v_0(t)$

$x_1 = i(t)$, $x_2 = v_0(t)$ — Peubah Keadaan

SISO.

Hukum Ohm:

(1) $v_i(t) - v_0(t) = Ri(t) + L\dfrac{di(t)}{dt}$

(2) $i(t) = C\dfrac{dv_0(t)}{dt}$