* Bagaimana solusi

$\dot{x} = Ax + Bu$, $x(0) = x_0$, $u = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}$

$\dot{x} = Ax(t) + Bu(t)$, $A[n \times n]$, $B = [n \times m]$

Misalkan $x(t) = e^{At} c(t)$, $c(t) = \begin{bmatrix} c_1(t) \\ c_2(t) \\ \vdots \\ c_n(t) \end{bmatrix}$

$\dot{x} = \frac{dx(t)}{dt} = \frac{d[e^{At}]}{dt} c(t) + [e^{At}] \frac{dc(t)}{dt}$

$\dot{x} = A \underbrace{e^{At} c(t)} + [e^{At}] \frac{dc(t)}{dt}$

$\dot{x} = A x(t) + [e^{At}] \frac{dc(t)}{dt}$

$\dot{x} = A x(t) + B u(t)$ $-$

$0 = [e^{At}] \frac{dc(t)}{dt} - Bu(t)$

$[e^{At}] \frac{dc(t)}{dt} = Bu(t)$

$\underbrace{[e^{At}]^{-1}[e^{At}]}_{I} \frac{dc(t)}{dt} = [e^{At}]^{-1} Bu(t)$

$\frac{dc(t)}{dt} = e^{-At} Bu(t)$

$c(t) = \int \underbrace{e^{-At}_{[n \times n]} B_{[n \times m]} u(t)_{[m \times 1]}}_{[n \times 1]} dt$

solusi $\dot{x} = Ax + Bu$ adalah solusi $\dot{x} = Ax$ [umum] ditambah solusi $\dot{x} = Ax + Bu$

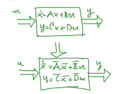
$$x(t) = x_0 e^{At} + \int e^{-At} Bu(t)\, dt$$

$y = Cx + Du$

Keluaran:

$$y(t) = C\left[x_0 e^{At} + \int e^{-At} Bu(t)\, dt\right] + Du(t)$$

* Transformasi SIMILARITAS



$\bar{x} = Tx$

T : matrix non singular, invertible, $\det(T) \neq 0$

Jika matrix A mempunyai nilai-eigen $\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix}$ maka matrix $\bar{A} = TAT^{-1}$ juga mempunyai nilai eigen yang sama

Oleh karena itu, matrix A akan "similar" dengan matrix diagonal

$A_d = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \ \lambda_n \end{bmatrix}$ karena ada matrix T sehingga $A = T A_d T^{-1}$

Dari solusi pers differensial:

$y(t) = C\left[\bar{x}_0 e^{A_d t} + \int e^{-At} Bu(t)\, dt\right]$

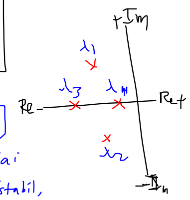
untuk $u(t) = 0$ (tanpa masukan) $+ Du(t)$

$y(t) = C\bar{x}_0 e^{A_d t} = C\bar{x}_0 \left[I + A_d t + \frac{1}{2} A_d^2 t^2 + \dots\right]$

$= C\bar{x}_0 \begin{bmatrix} 1 + \lambda_1 t + \frac{1}{2}\lambda_1^2 t^2 + \dots & 0 & \dots & 0 \\ 0 & 1 + \lambda_2 t + \frac{1}{2}\lambda_2^2 t^2 + \dots & 0 & 0 \\ 0 & 0 & 1 + \lambda_3 t + \frac{1}{2}\lambda_3^2 t^2 + \dots & \\ & & & \ddots \end{bmatrix}$

$= C\bar{x}_0 \begin{bmatrix} e^{\lambda_1 t} & & & 0 \\ & e^{\lambda_2 t} & & \\ & & e^{\lambda_3 t} & \\ 0 & & & \ddots \ e^{\lambda_n t} \end{bmatrix}$

Jika $\lambda_1, \lambda_2, \lambda_3, \dots \lambda_n$ semuanya mempunyai bagian REAL yang negatif, $e^{\lambda_1 t} = e^{(a+jb)t}$, $a < 0$ maka $e^{\lambda_i t}$ menuju nol untuk $t \to \infty$, artinya sistem STABIL



Jika ada salah satu saja dari nilai eigen matrix A bagian real-nya non-negatif, maka sistem tidak stabil, karena nilai eigen tersebut akan membuat $e^{\lambda t}$ tidak menuju nol untuk $t \to \infty$

Contoh

* stabilkah

$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

* Jawab

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow |\lambda I - A| = 0$

$\left|\begin{bmatrix} \lambda & -1 \\ 0 & \lambda \end{bmatrix}\right| = 0$

$\lambda^2 = 0 \rightarrow \lambda_1 = \lambda_2 = 0$

(tidak stabil)

* stabilkah sistem suspensi tanpa shockbreaker $(B = 0)$?

$A = \begin{bmatrix} 0 & -\frac{K}{M} \\ 1 & 0 \end{bmatrix}$ $[\lambda I - A] = \begin{bmatrix} \lambda & \frac{K}{M} \\ -1 & \lambda \end{bmatrix}$

$\det[\lambda I - A] = \lambda^2 + \frac{K}{M} = 0$

$\lambda_{1,2} = \pm j\sqrt{\frac{K}{M}}$, non-negatif

Bagaimana jika tanpa pegas $(K = 0)$

TIDAK STABIL

* Stabilkan sistem pemuatan C tanpa R !

$A = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}$ $[\lambda I - A] = \begin{bmatrix} \lambda & \frac{1}{L} \\ -\frac{1}{C} & \lambda \end{bmatrix}$

$\det[\lambda I - A] = 0$

$\lambda^2 + \frac{1}{LC} = 0 \rightarrow \lambda^2 = -\frac{1}{LC}$

$\lambda_{1,2} = \pm j\sqrt{\frac{1}{LC}}$, non-negatif

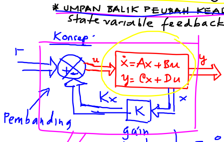
Bagaimana jika tanpa induktor $(L = 0)$

TIDAK STABIL

* UMPAN BALIK PEUBAH KEADAAN

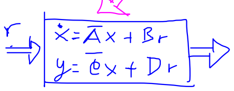
State variable feedback

Konsep:



$u = r - Kx$, $u\,[m \times 1]$, $r\,[m \times 1]$, $Kx\,[n \times 1]$

gain matrix, $K[m \times n]$



$\dot{x} = Ax + B[r - Kx]$

$= Ax + Br - BKx$

$= [A - BK]x + Br$

$= \bar{A}x + Br$

$y = Cx + Du$

$= Cx + D[r - Kx]$

$= Cx + Dr - DKx$

$= [C - DK]x + Dr$

$= \bar{C}x + Dr$

$\bar{A} = A - BK$

$\bar{C} = C - DK$

Nilai eigen $\bar{A}$ dapat ditempatkan di mana saja dalam bidang kompleks dengan mengatur K

Contoh:

$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

adalah sistem yang tak stabil, karena nilai-eigen matrix A $\lambda_1 = \lambda_2 = 0$ (tidak negatif bagian Real-nya)

Untuk menstabilkan sistem di atas maka digunakan umpan balik peubah keadaan dengan gain matrix $K = [K_1 \ K_2]$

$m = 1$, $n = 2$, $K[m \times n]$, $K = [1 \times 2]$

$\bar{A} = A - BK = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} K_1 & K_2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K_1 & -K_2 \end{bmatrix}$

Nilai eigen $\bar{A}$, $\bar{\lambda}_1$ dan $\bar{\lambda}_2$ dapat dihitung sebagai berikut:

$[\bar{\lambda} I - \bar{A}] = \begin{bmatrix} \bar{\lambda} & -1 \\ K_1 & \bar{\lambda} + K_2 \end{bmatrix} \rightarrow \det[\bar{\lambda} I - \bar{A}] = 0$

$\bar{\lambda}(\bar{\lambda} + K_2) + K_1 = 0$

$\bar{\lambda}^2 + K_2\bar{\lambda} + K_1 = 0$

Misalnya diinginkan $\bar{\lambda}_1 = \bar{\lambda}_2 = -1$ (critically damped, teredam kritis)

$(\bar{\lambda} + 1)^2 = 0$

$\bar{\lambda}^2 + 2\bar{\lambda} + 1 = 0$

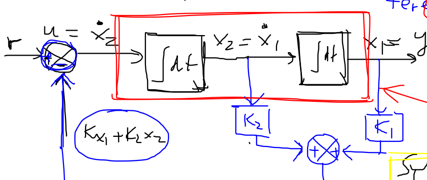
$\bar{\lambda}^2 + K_2\bar{\lambda} + K_1 = 0$

$K_1 = 1$, $K_2 = 2$ $\Rightarrow$ $\bar{\lambda}_1 = \bar{\lambda}_2 = -1$ (stabil, teredam kritis)

NEXT:

MID Bahan sampai di sini

Bagan Kotak.



Syarat kestabilan

$K_1 > 0$ $K_2 > 0$