$\int x\,dx = \frac{1}{2}x^2$

$\int x^2 dx = \frac{1}{3}x^3$

$\int x^{-2} dx = \frac{1}{x}$

$\int x^{-1} dx = \ln x$

$\int \frac{dx}{x} = \ln x$

*** Solusi Persamaan Keadaan**

Per Differensial

$\frac{dx(t)}{dt} = a\,x(t)$, $x(0) = x_0$ (kondisi awal)

$x(t) = \ldots\ldots ???$ (solusi) a = konstanta

Solusi:

$\int \frac{dx(t)}{x(t)} = \int a\,dt = a\int dt = at + C$

$\ln[x(t)] = at + C$

$e^{\log[x(t)]} = e^{at+C} \longrightarrow x(t) = e^{[at+C]} = e^{C}\cdot e^{at} = Ae^{at}$

$x(0) = Ae^{0} = A = x_0$

Solusi: $\boxed{x(t) = x_0 e^{at}}$

verifikasi: $\frac{dx(t)}{dt} = x_0 a e^{at} = a[x_0 e^{at}] = a\,x(t)$ ok

$\frac{d[e^{at}]}{dt} = ae^{at}$ — ???

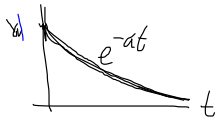
$u = at$ $\quad \frac{du}{dt} = a \longrightarrow \frac{d[e^u]}{du}, \frac{du}{dt} = e^u \cdot a = ae^{at}$

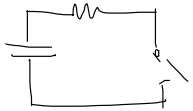
$\frac{d[e^u]}{du} = e^u$ — ??

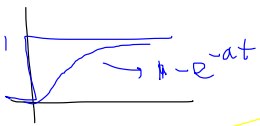
$e^u = 1 + u + \frac{1}{2}u^2 + \frac{1}{6}u^3 + \frac{1}{24}u^4 + \frac{1}{120}u^5 + \ldots + \frac{1}{n!}u^n + \ldots$

$\frac{de^u}{du} = 0 + \underbrace{1 + u + \frac{1}{2}u^2 + \frac{1}{6}u^3 + \frac{1}{24}u^4 + \ldots + \frac{1}{(n-1)!}u^{(n-1)}}_{e^u}$

$\Longrightarrow e = e^1 = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \ldots + \frac{1}{n!} + \ldots$







Pers. Differensial:

$\dot{x} = Ax$

$x(0) = x_0$

$x = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$ $\quad A[n \times n]$

$\dot{x} = \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \vdots \\ \frac{dx_n(t)}{dt} \end{bmatrix}$ $\quad x_0 = \begin{bmatrix} x_{01} \\ x_{02} \\ \vdots \\ x_{0n} \end{bmatrix}$

Solusi: $x(t) = x_0[e^{At}]$

$[e^{At}] = \ldots\ldots ???$

$[e^{At}]$ matrix transisi $[n \times n]$

$$[e^{At}] = \left[I + At + \frac{1}{2}A^2t^2 + \frac{1}{6}A^3t^3 + \frac{1}{24}A^4t^4 + \ldots\right]$$

$$x(t) = x_0\left[I + At + \frac{1}{2}A^2t^2 + \frac{1}{6}A^3t^3 + \frac{1}{24}A^4t^4 + \ldots\right]$$

$$\frac{dx(t)}{dt} = x_0\left[A + A^2t + \frac{1}{2}A^3t^2 + \frac{1}{6}A^4t^3 + \ldots\right]$$

$$= x_0 A\left[I + At + \frac{1}{2}A^2t^2 + \frac{1}{6}A^3t^3 + \ldots\right]$$

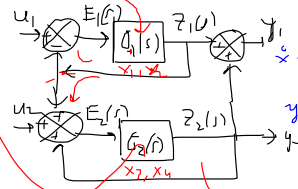
$$= \underbrace{Ax_0\left[I + At + \frac{1}{2}A^2t^2 + \frac{1}{6}A^3t^3 + \ldots\right]}$$

$$= Ax(t) \quad \text{(terbukti)}$$

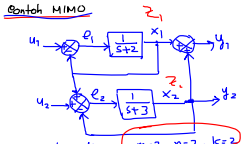
$$\dot{x} = Ax$$

$\xi_1(s) = \frac{1}{s^2+2s+1} = \frac{Z_1(s)}{E_1(s)}$

x_1, x_2



Contoh MIMO



$\frac{X_1(s)}{E_1(s)} = \frac{1}{s+2}$ kasus $m<n$, $m=0$, b_0, $n=1$, $a_0=2$, $a_1=1$

$\dot{x}_1 = [-2]x_1 + [1]e_1$

$-\frac{a_0}{a_1}$ $\quad \frac{1}{a_1}$

$\dot{x}_1 = -2x_1 + e_1$

$= -2x_1 + [-y_1 + u_1]$

$\dot{x}_1 = -3x_1 + u_1$

$\frac{X_2(s)}{E_2(s)} = \frac{1}{s+3}$

$\dot{x}_2 = -3x_2 + e_2$

$\dot{x}_2 = -3x_2 + [x_1 - x_2 + u_2]$

$\dot{x}_2 = -4x_2 + x_1 + u_2$

Suatu sistem dengan $m=2, n=2, k=2$

Susunlah model Ruang Keadaannya!

$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$ $\quad A[2\times2]$ $B[2\times2]$ $C[2\times2]$ $D[2\times2]$

$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -3 & 0 \\ 1 & -4 \end{bmatrix}}_{[A]}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{[B]}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{[C]}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{[D]}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \bigcirc & \bigcirc \\ \bigcirc & \bigcirc \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \\ \end{bmatrix}$