Pers. Keluaran : $y = Cx + Du$

Transf. Laplace : $Y(s) = C X(s) + D U(s)$

$= C[sI-A]^{-1} B\, U(s) + D U(s)$

$= [\underbrace{\underbrace{C}_{[1\times n]} \underbrace{[sI-A]^{-1}}_{[n\times n]} \underbrace{B}_{[n\times 1]}}_{[1\times 1]} + D]\, U(s)$ $[1\times 1]$

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = [C[sI-A]^{-1}B + D]}$$

(1) $\dfrac{di(t)}{dt} = -\dfrac{R}{L} i(t) - \dfrac{1}{L} v_0(t) + \dfrac{1}{L} v_i(t)$

$\dot{x}_1 = -\dfrac{R}{L} x_1 - \dfrac{1}{L} x_2 + \dfrac{1}{L} u$

(2) $\dfrac{dv_0(t)}{dt} = \dfrac{1}{C} i(t)$

$\dot{x}_2 = \dfrac{1}{C} x_1$

$m=1$, $n=2$, $k=1$

$A[2\times 2]$, $B[2\times 1]$, $C[1\times 2]$, $D[1\times 1]$

Pers. keadaan : $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-\frac{R}{L} & -\frac{1}{L}\\ \frac{1}{C} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\frac{1}{L}\\ 0\end{bmatrix} u$

Pers. keluaran $y = [0 \quad 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix} + [0]\, u$

Hubungan antara Model Nisbah Alih dgn Model Ruang Keadaan

* Model Ruang Keadaan bersifat lebih umum (general) dari Model Nisbah Alih
* Semua model Nisbah Alih dapat diubah menjadi model Ruang Keadaan, tapi belum tentu sebaliknya
* Model Ruang Keadaan yang dapat diubah menjadi model Nisbah Alih, hanyalah model dari sistem SISO, Linear, Time Invariant

* Model Ruang Keadaan $\longrightarrow$ Nisbah Alih

Perintah ; >>> ss2tf

$U(s) \rightarrow \boxed{G(s)} \rightarrow Y(s)$

Model Ruang Keadaan :

$\begin{bmatrix}\dot{x} = Ax + Bu\\ y = Cx + Du\end{bmatrix} \Rightarrow G(s) = \dfrac{Y(s)}{U(s)}$

$m=1$, $n\geq 1$, $k=1$

$u = u(t) = \mathcal{L}^{-1} U(s)$

$y = y(t) = \mathcal{L}^{-1} Y(s)$

$A[n\times n]$, $B[n\times 1]$, $C[1\times n]$, $D[1\times 1]$

Pers. Keadaan : $\dot{x} = Ax + Bu$

$\dfrac{dx(t)}{dt} = A\,x(t) + B\,u(t)$

Transf. Laplace : $sX(s) = A\,X(s) + B\,U(s)$

$\underbrace{sX(s) - AX(s)}$ $= B\,U(s)$

$sI = \begin{bmatrix}s & & 0\\ & s & \\ 0 & & s\end{bmatrix}$ $[n\times n]$

$[sI - A]\,X(s) = B\,U(s)$

$\underbrace{[sI-A]^{-1}[sI-A]}_{I} X(s) = [sI-A]^{-1} B\,U(s)$

$s\begin{bmatrix}1 & & 0\\ & 1 & \\ 0 & & 1\end{bmatrix}$

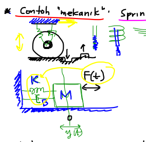
$X(s) = [sI-A]^{-1} B\,U(s)$

Contoh²

* Contoh "matematis" Tentukan dimensi matrix A, B, C dan D dari sebuah sistem yang punya 3 masukan, 4 peubah keadaan dan 5 keluaran

Jawab $m=3$, $n=4$, $k=5$ $\rightarrow$ $A[4\times 4]$ $C[5\times 4]$ $B[4\times 3]$ $D[5\times 3]$

* Contoh "mekanik" : Spring-Mass-Damper Systems

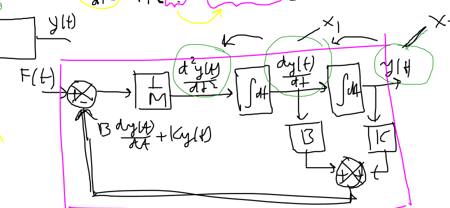


$F(t) = M\dfrac{d^2y(t)}{dt^2} + B\dfrac{dy(t)}{dt} + K\,y(t)$

percepatan kecepatan posisi

$\dfrac{d^2y(t)}{dt^2} = \dfrac{1}{M}\left\{F(t) - \left[B\dfrac{dy(t)}{dt} + Ky(t)\right]\right\}$

$F(t) \longrightarrow \boxed{\quad} \longrightarrow y(t)$



Ditanyakan : Model RUANG KEADAAN sistem di atas !

* SISO $\longrightarrow$ $m=1$, $k=1$, $n=2$

$u \triangleq F(t)$

$y \triangleq y(t)$

$A[2\times 2]$ $C[1\times 2]$

$B[2\times 1]$ $D[1\times 1]$

$x = \underbrace{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}_{\text{2 peubah keadaan}} = \begin{bmatrix}\frac{dy(t)}{dt}\\ y(t)\end{bmatrix}$ $\begin{matrix}\rightarrow \text{kecepatan}\\ \rightarrow \text{posisi}\end{matrix}$

Model Ruang Keadaannya :

$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \dot{x} = \underbrace{\begin{bmatrix}-\frac{B}{M} & -\frac{K}{M}\\ 1 & 0\end{bmatrix}}_{A[2\times 2]}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}\frac{1}{M}\\ 0\end{bmatrix}}_{B[2\times 1]} u$

$y = \underbrace{[0 \quad 1]}_{C[1\times 2]}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{[0]}_{D[1\times 1]}\, u$

$\dfrac{d^2y(t)}{dt^2} = \dot{x}_1 = \dfrac{1}{M}\left[F(t) - \left[B\dfrac{dy(t)}{dt} + Ky(t)\right]\right]$

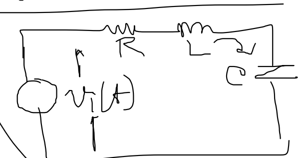
$= \dfrac{1}{M}\left[u - Bx_1 - Kx_2\right]$

$= -\dfrac{B}{M}x_1 - \dfrac{K}{M}x_2 + \dfrac{1}{M}u$

$\dfrac{dy(t)}{dt} = \dot{x}_2 = x_1$

* Contoh Elektrik :

Pemuatan Kapasitor melalui rangkaian RLC serie



$u \rightarrow \boxed{\begin{matrix}\dot{x} = Ax + Bu\\ y = Cx + Du\end{matrix}} \rightarrow y$

Masukan : $u \triangleq v_i(t)$

Keluaran : $y \triangleq v_0(t)$

$\left.\begin{matrix}x_1 = i(t)\\ x_2 = v_0(t)\end{matrix}\right\}$ Peubah Keadaan

SISO .

Hukum Ohm :

(1) $v_i(t) - v_0(t) = R\,i(t) + L\dfrac{di(t)}{dt}$

(2) $i(t) = C\dfrac{dv_0(t)}{dt}$

*** Contoh Electrik**

$R = 100\,\text{Ohm}$ $C = 0{,}1\,F$ $L = 0{,}01\,H$

Model Ruang Keadaan:

$u = v_i(t)$ $y = v_o(t)$

$x_1 = i(t)$ $x_2 = v_o(t)$

$$A = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -1000 & -100 \\ 10 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = [0]$$

$$G(s) = C[sI - A]^{-1}B + D$$

$$[sI - A] = \begin{bmatrix} s+1000 & 100 \\ -10 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\text{adj}\,[sI - A]}{\det[sI - A]}$$

$$= \frac{1}{s^2 + 1000s + 1000} \begin{bmatrix} s & -100 \\ 10 & s+1000 \end{bmatrix}$$

$$G(s) = \frac{1}{s^2 + 1000s + 1000} \left[\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s & -100 \\ 10 & s+1000 \end{bmatrix} \begin{bmatrix} 100 \\ 0 \end{bmatrix} \right].$$

$$= \frac{1}{s^2 + 1000s + 1000} \begin{bmatrix} 10 & s+1000 \end{bmatrix} \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$= \frac{1000}{s^2 + 1000s + 1000}$$

check. $G(s) = \dfrac{1/LC}{s^2 + \frac{R}{L}s + 1/LC}$

$$= \frac{1}{LCs^2 + RCs + 1}$$

Contoh ss2tf

*** Contoh Mekanik**

Model Ruang Keadaan

$u = F(t)$

$y = y(t)$

$x_1 = \dfrac{dy(t)}{dt}$ $x_2 = y(t)$

$$A = \begin{bmatrix} -\frac{B}{M} & -\frac{K}{M} \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{M} \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = [0]$$

Spring Mass Damper

$M = 2\,kg$

$B = 4\ N\,sec/m$

$= 4\ \dfrac{kg\,\cancel{m}}{sec^2}\ \dfrac{\cancel{sec}}{\cancel{m}}$

$= 4\ kg/sec$

$K = 6\ N/m = 6\ kg/sec^2$

$$A = \begin{bmatrix} -2 & -3 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = [0],$$

$$G(s) = C[sI - A]^{-1}B + D$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -3 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} s+2 & 3 \\ -1 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\text{adj}\,[sI - A]}{\det[sI - A]}$$

$$\text{adj}\,[sI - A] = \text{adj} \begin{bmatrix} s+2 & 3 \\ -1 & s \end{bmatrix} = \begin{bmatrix} s & -3 \\ 1 & s+2 \end{bmatrix}$$

$$\begin{bmatrix} s+2 & 3 \\ -1 & s \end{bmatrix} \times \begin{bmatrix} s & -3 \\ 1 & s+2 \end{bmatrix} = \frac{\begin{bmatrix} \cancel{s^2+2s+3} & 0 \\ 0 & s^2\cancel{+2s+3} \end{bmatrix}}{s^2+2s+3}$$

$$\det[sI - A] = s(s+2) + 3 = s^2 + 2s + 3$$

$$G(s) = \frac{1}{s^2 + 2s + 3} \left[\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s & -3 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} \right] = \frac{1}{s^2+2s+3} \begin{bmatrix} 1 & s+2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}$$

$$= \frac{1/2}{s^2 + 2s + 3} = \frac{1}{2s^2 + 4s + 6}$$

$$= \frac{1}{Ms^2 + Bs + K}$$

Model Nisbah Alih (transfer function) $\Rightarrow$

*** tf2ss**

Semua model Nisbah Alih (transfer function) pasti dapat diubah menjadi model Ruang Keadaan (state space)

Dari model Nisbah Alih suatu sistem yang sama dapat dibuat bermacam-macam model Ruang Keadaan yang ber-beda2 yang semuanya sahih (valid)

Berikut ini akan diberikan satu contoh prosedur tf2ss yang menghasilkan matrix A dengan bentuk tertentu yang disebut matrix "Kawanan Jordan" (Jordan Companion Matrix)

$$G(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_1 s + a_0}$$

$$= \frac{N(s)}{D(s)}$$

$N(s) \longrightarrow$ pembilang (Numerator)

$D(s) \longrightarrow$ penyebut (Denominator)

$N(s)$ polynomial s berderajat m dengan koefisien Real $b_0, b_1, \dots, b_m$

$D(s)$: polynomial s berderajat n dengan koefisien Real $a_0, a_1, \dots, a_n$

$m \le n$ (tidak dikenal kasus $m > n$) HPF

- Kasus $m < n$
- Kasus $m = n$

next'