

$$x(t) = X_0 \left(1 + at + \frac{1}{2} a^2 t^2 + \frac{1}{6} a^3 t^3 + \frac{1}{24} a^4 t^4 + \dots + \frac{1}{n!} a^n t^n \right)$$

Check:

$$* t=0 \rightarrow x(0) = X_0 \quad \checkmark$$

$$\begin{aligned} * \frac{dx(t)}{dt} &= X_0 \left(a + a^2 t + \frac{1}{2} a^3 t^2 + \frac{1}{6} a^4 t^3 + \dots + \frac{1}{(n-1)!} a^n t^{n-1} \right) \\ &= a X_0 \left(1 + at + \frac{1}{2} a^2 t^2 + \frac{1}{6} a^3 t^3 + \dots + \frac{1}{(n-1)!} a^{n-1} t^{n-1} \right) \\ &\quad \underbrace{\hspace{15em}}_{x(t)} \\ &= a x(t) \quad \checkmark \end{aligned}$$

Artinya: $\underline{e^{at} = 1 + at + \frac{1}{2} a^2 t^2 + \frac{1}{6} a^3 t^3 + \dots}$

$$e = e^1 = 1 + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \dots$$