

328D4103
Sistem Kendali + Praktikum
MODUL 06
Keterkendalian dan
Keteramatan
(Semester Awal 2020-2021)



Keterkendalian dan Keteramatan

- Sumber pembelajaran: <https://web.unhas.ac.id/rhiza/arsip/kuliah/Sistem-Kendali/Catatan-Kuliah-2017/>
- Catatan Kuliah Lengkap 2016-2017: <https://web.unhas.ac.id/rhiza/arsip/kuliah/Sistem-Kendali/Catatan-Kuliah-Sistem-Kendali-2016-2017.pdf> (dibuka *on-line* saja, tidak perlu diunduh)



DEFINISI

- Suatu kendalian yang dimodelkan dengan Model Ruang Keadaan, dikatakan mempunyai bagian yang **TERKENDALI** (**CONTROLLABLE**) jika semua peubah keadaan pada bagian itu dapat dipengaruhi oleh minimal salah satu isyarat masukan.
- Suatu kendalian yang dimodelkan dengan Model Ruang Keadaan, dikatakan mempunyai bagian yang **TERAMATI** (**OBSERVABLE**) jika semua peubah keadaan pada bagian itu dapat mempengaruhi minimal salah satu isyarat keluaran.
- Jika semua bagian dari suatu kendalian yang dimodelkan dengan Model Ruang Kendalian merupakan bagian yang **TERKENDALI** (**CONTROLLABLE**), maka kendalian tersebut dikatakan **SEPENUHNYA TERKENDALI** (**COMPLETELY CONTROLLABLE**).
- Jika semua bagian dari suatu kendalian yang dimodelkan dengan Model Ruang Kendalian merupakan bagian yang **TERAMATI** (**OBSERVABLE**), maka kendalian tersebut dikatakan **SEPENUHNYA TERAMATI** (**COMPLETELY OBSERVABLE**).

KONSEKUENSI

- Suatu kendalian yang **SEPENUHNYA TERKENDALI** (**COMPLETELY CONTROLLABLE**) pasti dapat distabilkan, dikatakan sebagai kendalian yang *stabilizable*.
- Suatu kendalian yang **SEPENUHNYA TERKENDALI** (**COMPLETELY CONTROLLABLE**) dan **SEPENUHNYA TERAMATI** (**COMPLETELY OBSERVABLE**) pasti dapat distabilkan dengan pengendali umpan-balik peubah keadaan (*state-variable feedback*).
- Suatu kendalian yang **SEPENUHNYA TERKENDALI** (**COMPLETELY CONTROLLABLE**) tapi **TIDAK SEPENUHNYA TERAMATI** (**NOT COMPLETELY OBSERVABLE**) masih dapat distabilkan dengan pengendali umpan-balik peubah keadaan (*state-variable feedback*), dengan menambahkan sistem pengamat atau *observer*.
- Suatu kendalian yang **TIDAK SEPENUHNYA TERKENDALI** (**NOT COMPLETELY CONTROLLABLE**) masih dapat distabilkan, asalkan bagian yang tidak stabil merupakan bagian yang **TERKENDALI** (**CONTROLLABLE**).

CONTOH (dikutip dari Friedland, [1986], page 191-193)

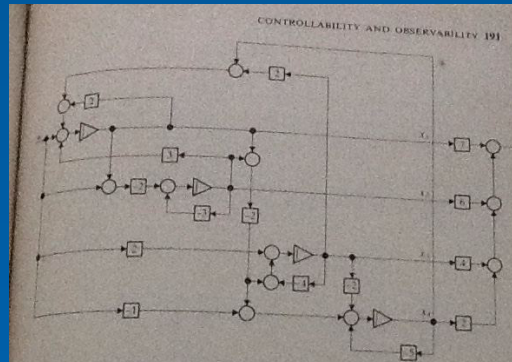


Figure 5.1 Hypothetical fourth-order system to illustrate controllability and observability.

The differential equations of the system, obtained by inspection of the block diagram are

$$\begin{aligned} \dot{x}_1 &= 2x_1 + 3x_2 + 2x_3 + x_4 + u \\ \dot{x}_2 &= -2x_1 - 3x_2 - 2u \\ \dot{x}_3 &= -2x_1 - 2x_2 - 4x_3 + 2u \\ \dot{x}_4 &= -2x_1 - 2x_2 - 2x_3 - 5x_4 - u \end{aligned} \quad (5A.1)$$

and the observation equation is

$$y = 7x_1 + 6x_2 + 4x_3 + 2x_4 \quad (5A.2)$$

Thus, the matrices of the state-space representation are

$$A = \begin{bmatrix} 2 & 3 & 2 & 1 \\ -2 & -3 & 0 & 0 \\ -2 & -2 & -4 & 0 \\ -2 & -2 & -2 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -2 \\ 2 \\ -1 \end{bmatrix} \quad C = \begin{bmatrix} 7 & 6 & 4 & 2 \end{bmatrix}$$

The resolvent corresponding to A is given by

$$(sI - A)^{-1} = \begin{bmatrix} s^4 + 12s^3 + 47s^2 + 6s & 3s^3 + 21s + 36 & 2s^3 + 14s + 24 & s^3 + 7s + 12 \\ -2s^4 - 18s^2 - 40 & s^4 + 7s^3 + 8s - 16 & 4s - 16 & -2s - 8 \\ -2s^4 - 12s^2 - 10 & -2s^4 - 12s - 10 & s^4 + 6s^3 + 7s + 2 & -2s - 2 \\ -2s^4 - 6s - 4 & -2s^4 - 6s - 4 & -2s^4 - 6s - 4 & s^4 + 5s^3 + 9s + 4 \end{bmatrix}$$

Then the transfer function from the input u to the output y is given by

$$M(s) = C(sI - A)^{-1}B = \frac{2s^4 + 14s^2 + 24}{(s+1)(s+2)(s+3)(s+4)}$$

which is the ratio of a 4th-degree polynomial to a 4th-degree polynomial. Notice the numerator is the same as the denominator. However, we discover that

$$M(s) = \frac{2(s+1)(s+2)(s+3)(s+4)}{(s+1)(s+2)(s+3)(s+4)} = 2$$

Thus, three of the poles (at $s = -1, -2, -3$) are cancelled by zeros at exactly the same locations and what seems to be a fourth-order transfer function is actually only first order. To fully explain this rather remarkable behavior, the following change of state variables is performed:

$$x = Tx$$

when

$$T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix} \quad \text{and} \quad T^{-1} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The matrix T happens to be a diagonalizing transformation

$$TAT^{-1} = \Lambda = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

and the corresponding control and observation matrices are

$$B = TB^{-1} = \begin{bmatrix} 1 \\ -2 \\ 2 \\ -1 \end{bmatrix} \quad C = CT^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}$$

Hence the corresponding state equations are

$$\begin{aligned} \dot{x}_1 &= -x_1 + u \\ \dot{x}_2 &= -2x_2 \\ \dot{x}_3 &= -3x_3 + u \\ \dot{x}_4 &= -4x_4 \end{aligned} \quad (5A.3)$$

and the observation equation is

$$y = x_1 + x_2 \quad (5A.4)$$

A block diagram representation of (5A.3) and (5A.4) is shown in Fig. 5.2. Clearly, the input u affects only the state variables x_1 and x_3 , and x_2 and x_4 are unaffected by the input. The output y depends only on x_1 and x_2 , and x_3 and x_4 do not contribute to the output. Thus, in the transformed coordinates, the system has four different subsystems. (In this case each subsystem is only first-order.)

- x_1 : affected by the input, visible in the output
- x_2 : unaffected by the input, visible in the output
- x_3 : affected by the input, invisible in the output
- x_4 : unaffected by the input, invisible in the output

Only the first subsystem x_1 contributes to the transfer function $M(s)$, which clearly is $1/(s+1)$.

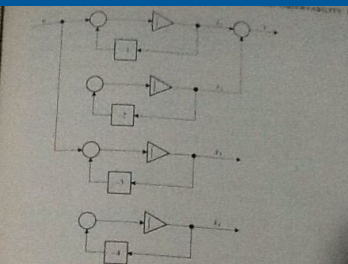


Figure 5.2 System equivalent of Fig. 5.1 showing separation into controllable and observable subsystems.

Example 5A is a microcosm of the general case. As Kalman has shown, every system of the generic form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

can be transformed into the four subsystems of Fig. 5.2. The first subsystem is both controllable and observable; the second is uncontrollable but observable; the third is controllable but unobservable; and the fourth is neither observable nor controllable. The transfer function of the system is determined only by the controllable and observable subsystem. It thus follows that if the transfer function of a single-input, single-output system is of lower degree than the dimension of the state-space, then the system must contain an uncontrollable subsystem, or an unobservable subsystem, or possibly both. By convention, if a system contains an uncontrollable subsystem it is said to be uncontrollable; likewise, if it contains an unobservable subsystem it is said to be unobservable. (See Note 5.2.)

The system in the foregoing example is asymptotically stable: all its poles are in the left half-plane, so the consequences of the system being uncontrollable and unobservable are innocuous. Any initial conditions on the uncontrollable

Contoh: Kendalian: $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$

$A = \begin{bmatrix} 2 & 3 & 2 & 1 \\ -2 & -3 & 0 & 0 \\ -2 & -2 & -4 & 0 \\ -2 & -2 & -2 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ -2 \\ 2 \\ -1 \end{bmatrix}$ $C = \begin{bmatrix} 7 & 6 & 4 & 2 \end{bmatrix}$ $D = \begin{bmatrix} 0 \end{bmatrix}$

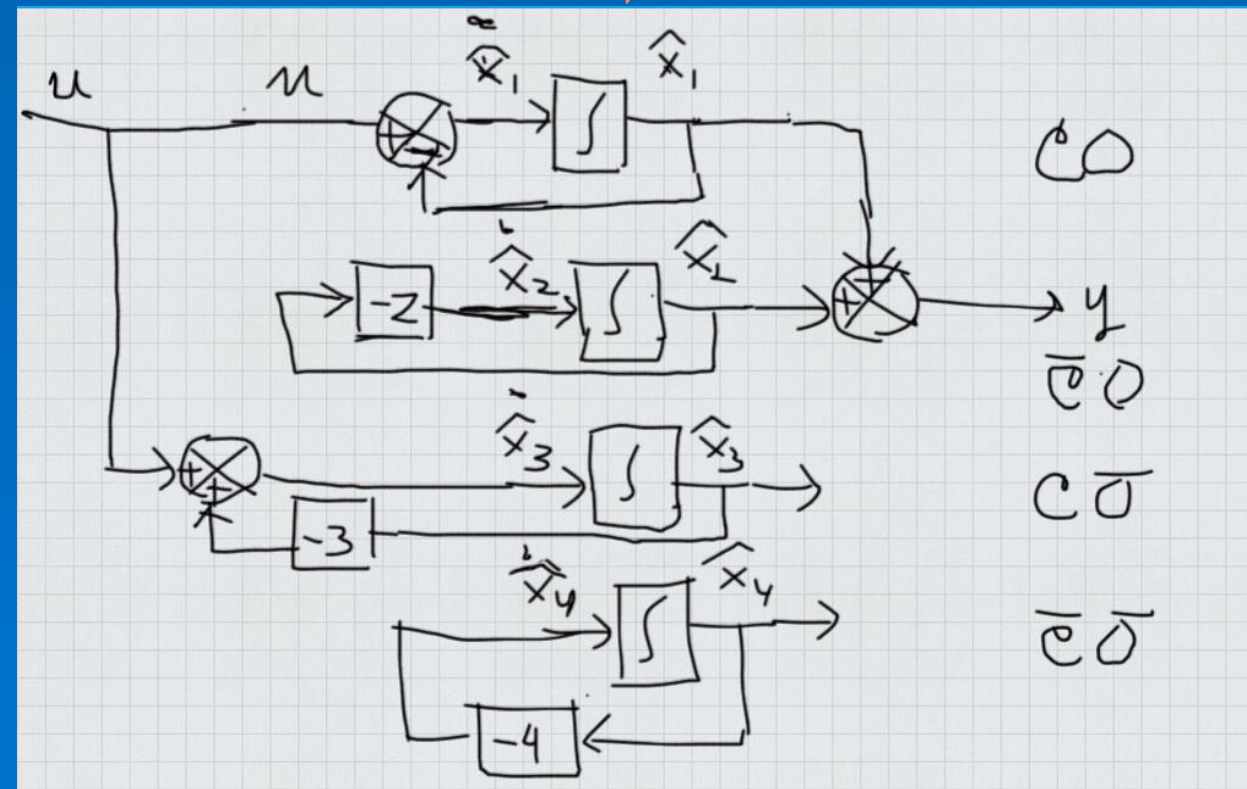
$T = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 3 & 3 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ $T^{-1} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$

Transf. SIMILARITAS

$\hat{x} = Tx$

$\hat{A} = TAT^{-1} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}$ $\hat{B} = TB = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$\hat{C} = CT^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}$



Pers. Keadaan: $\begin{cases} \dot{\hat{x}}_1 = -\hat{x}_1 + u \\ \dot{\hat{x}}_2 = -2\hat{x}_2 \\ \dot{\hat{x}}_3 = -3\hat{x}_3 + u \\ \dot{\hat{x}}_4 = -4\hat{x}_4 \end{cases} \quad [A], [B]$

$\hat{x} = Tx$

hasil Transf. SIMILARITAS

Pers. Keluaran: $y = \hat{x}_1 + \hat{x}_2 \quad [\hat{C}], [\hat{D}]$

$\hat{x}_1: CO$
 $\hat{x}_2: CO$
 $\hat{x}_3: CO$
 $\hat{x}_4: CO$

→ Bagan Kotak →
 nilai-eigen A dan $\hat{A}$
 $\lambda = \begin{bmatrix} -1 \\ -2 \\ -3 \\ -4 \end{bmatrix} \Rightarrow$ kendalian stabil

UJI KETERKENDALIAN

- **KETERKENDALIAN** dari suatu kendalian **LTI** (*Linear Time-Invariant*, matrix **A**, **B**, **C** dan **D** semua **matrix konstan**) yang dimodelkan dengan Model Ruang Keadaan dapat diuji dengan membentuk **Matrix Keterkendalian P**.
- Untuk matrix **A** [**n x n**] dan matrix **B** [**n x m**], **Matrix Keterkendalian P** [**n x [nxm]**] (**n** baris [**nxm**] kolom):

$$P = [B \quad AB \quad A^2B \quad A^3B \quad \dots \quad A^{n-1}B]$$

- Kendalian **LTI SEPENUHNYA TERKENDALI** (**COMPLETELY CONTROLLABLE**) jika dan hanya jika matrix keterkendalian **P**-nya **full rank** atau **rank(P) = n**
- Khusus untuk kendalian **LTI** yang **single-input**, **m = 1**, dengan matrix **P** [**n x n**], akan **SEPENUHNYA TERKENDALI** (**COMPLETELY CONTROLLABLE**), **rank(P) = n**, jika dan hanya jika matrix **P non-singular = invertible = determinannya tidak sama dengan nol**.

UJI KETERAMATAN

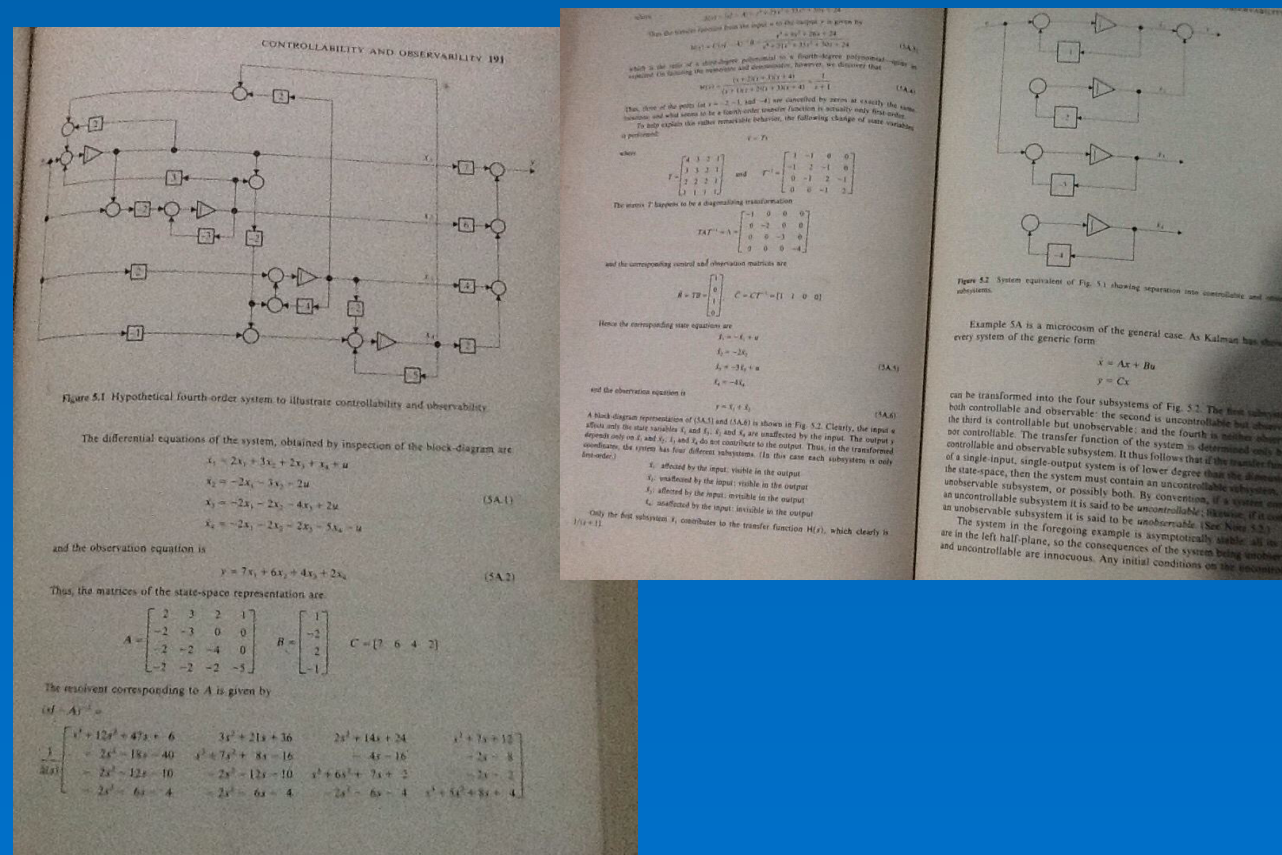
- **KETERAMATAN** dari suatu kendalian **LTI** (*Linear Time-Invariant*, matrix **A**, **B**, **C** dan **D** semua **matrix konstan**) yang dimodelkan dengan Model Ruang Keadaan dapat diuji dengan membentuk **Matrix Keteramatan Q**.
- Untuk matrix **A** [**n x n**] dan matrix **C** [**k x n**], **Matrix Keteramatan Q** [**n x [nxk]**] (**n** baris [**nxk**] kolom):

$$Q = [C^T \quad A^T C^T \quad (A^T)^2 C^T \quad (A^T)^3 C^T \quad \dots \quad (A^T)^{n-1} C^T]$$

- Kendalian **LTI SEPENUHNYA TERAMATI** (**COMPLETELY OBSERVABLE**) jika dan hanya jika matrix keterkendalian **Q**-nya **full rank** atau **rank(Q) = n**
- Khusus untuk kendalian **LTI** yang **single-output**, **k = 1**, dengan matrix **Q** [**n x n**], akan **SEPENUHNYA TERKENDALI** (**COMPLETELY OBSERVABLE**), **rank(Q) = n**, jika dan hanya jika matrix **Q non-singular = invertible = determinannya tidak sama dengan nol**.

TUGAS MANDIRI (tidak dikumpul)

- Silakan di-**Google** kata kunci: "**rank** dari suatu **matrix**", lalu dari referensi yang diperoleh, pelajari pengertian **rank** dari suatu matrix
- Tentukan matrix keterkendalian **P** dan matrix keteramatan **Q** dari contoh model kendalian **LTI** di bawah ini, dan juga hasil transformasi similaritas-nya (boleh menggunakan MATLAB)
- Tentukan **rank(P)** dan **rank(Q)** dari contoh di bawah ini, dan juga dari hasil transformasi similaritas-nya, dengan menggunakan perintah `>>> rank()` di MATLAB
- Apakah kendalian ini sepenuhnya terkendali dan sepenuhnya teramati? Jelaskan.



Contoh: Kendalian: $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$

$$A = \begin{bmatrix} 2 & 3 & 2 & 1 \\ -2 & -3 & 0 & 0 \\ -2 & -2 & -4 & 0 \\ -2 & -2 & -2 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -2 \\ 2 \\ -1 \end{bmatrix} \quad C = \begin{bmatrix} 7 & 6 & 4 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 3 & 3 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad T^{-1} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

Transf. SIMILARITAS

$$\hat{A} = TAT^{-1} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix} \quad \hat{B} = TB = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\hat{C} = CT^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}$$

MODUL PEMBELAJARAN SELANJUTNYA

- MODUL 01: (Pengantar/*Review*) Model RUANG KEADAAN (*State Space*)
- MODUL 02: Konversi Model RUANG KEADAAN ke NISBAH ALIH (ss2tf)
- MODUL 03: Konversi Model NISBAH ALIH ke RUANG KEADAAN (tf2ss)
- MODUL 04: Transformasi SIMILARITAS
- MODUL 05: TANGGAPAN dan KESTABILAN
- MODUL 06: KETERKENDALIAN dan KETERAMATAN
- **MODUL 07: UMPAN-BALIK PEUBAH KEADAAN**
- MODUL 08: PRAKTIKUM INDIVIDU & KELOMPOK (luring)

SELAMAT BELAJAR

Semoga SUKSES meraih PRESTASI!

