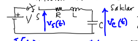
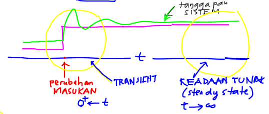
Analisis Transient

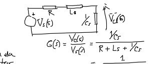
Contoh: Rangkaian Pemuatan Kapasitor C melalui R dan L seri



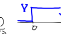
Saklar S ditutup pada t=0, kapasitor dimuati sampai penuh pada $t \to \infty$

Tentukan $G(s) = \frac{V_c(s)}{V_s(s)}$





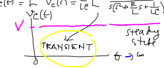
$$G(s) = \frac{V_c(s)}{V_s(s)} = \frac{1/Cs}{R + Ls + 1/Cs} = \frac{1}{LCs^2 + RCs + 1}$$



$$v_s(t) \to V_s(s) = \mathcal{L}\, v_s(t) = \frac{V}{s}$$

$$V_c(s) = G(s)\,V_s(s) = \frac{V}{s(LCs^2 + RCs + 1)}$$

$$v_c(t) = \mathcal{L}^{-1} V_c(s) = \frac{V}{LC}\mathcal{L}^{-1}\frac{1}{s(s^2 + \frac{R}{L}s + \frac{1}{LC})} \to \text{lihat tabel}$$

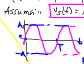


*Analisis Keadaan Tunak

Steady state analysis

$v_s(t) \to$ isyarat periodik

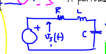
Assumsi: $v_s(t) = A_i \sin \omega t$



"omega" $\omega = 2\pi f$ [rad/sec]

f [Hertz] $= \frac{1}{T}$

T = perioda [second]



$v_c(t) = ???$

$v_c(t) = A_0 \sin(\omega t + \phi)$

Pada keadaan tunak (steady state)

Peubah Laplace $s \to j\omega$ $\quad j = \sqrt{-1}$ $\quad \omega = 2\pi f = \frac{2\pi}{T}$

$$G(s) = \frac{1}{LCs^2 + RCs + 1} \to G(j\omega) = \frac{1}{LC(j\omega)^2 + RCj\omega + 1} = \frac{1}{\underset{Im}{j\omega RC} + \underset{Re}{(1 - \omega^2 LC)}}$$

$$\text{Gain}\cdot A_v = |G(j\omega)| = \frac{A_0}{A_i} = \frac{1}{\sqrt{(\omega RC)^2 + (1 - \omega^2 LC)^2}}$$

(low pass filter)

$$\text{sudut fasa } \phi = -\tan^{-1}\frac{\omega RC}{1 - \omega^2 LC}$$

Bab I Pengenalan SISTEM LINIER

Bab II Pemodelan SISTEM LINIER

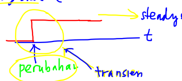
* Urgensi Pemodelan SISTEM ✓
* Pemodelan STATIK ⟹ Watak Alih (Transfer Characteristic)
* Pemodelan DINAMIK
  * Pemodelan Nisbah Alih (Transfer Function)
  * Pemodelan Ruang Keadaan (state space)

* Pemodelan STATIK

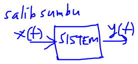
⇒ Pemodelan Watak Alih (Transfer Characteristics)

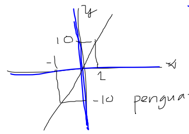
⇒ "Statik", tidak melibatkan parameter waktu t

⇒ "steady-state" (keadaan tunak) × "transien"

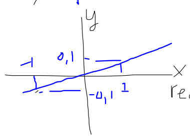


* Digambarkan dalam bentuk salib sumbu

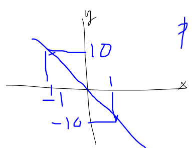




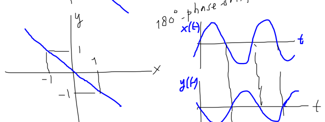
penguat (amplifier)



redaman (attenuator)

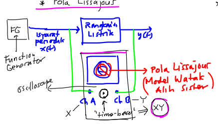


penguat membalik
inverting-amplifier



180° phase shifter

* Pola Lissajous

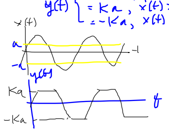


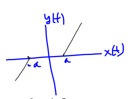
* Lebih bermanfaat untuk model sistem tak linier

→ Misalnya

Penguat jenuh (saturating amplifier)

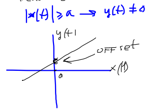
$$y(t)\begin{cases} = Kx(t), & |x(t)| \le a \\ = Ka, & x(t) > a \\ = -Ka, & x(t) < -a \end{cases}$$





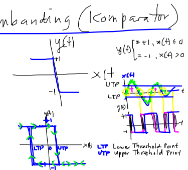
Dead Zone

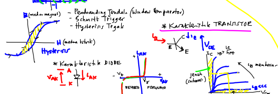
$|x(t)| \ge a \to y(t) \ne 0$



* Pembanding (Komparator)

$$y(t)\begin{cases} = +1, & x(t) \le 0 \\ = -1, & x(t) > 0 \end{cases}$$





* Liniersasikan penyearah (rectifier) $y(t) = |x(t)|$ pada titik kerja Q(0,0) dengan pendekatan garis singgung

MIDTEST

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14/04 } LIBUR

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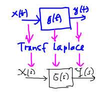
* Transformasi Laplace



~~$y(t) = g(t)\,x(t)$~~

t = time peubah REAL } time domain

$X(s) = \mathcal{L}\,x(t) \rightleftharpoons x(t) = \mathcal{L}^{-1}X(s)$

$Y(s) = \mathcal{L}\,y(t) \rightleftharpoons y(t) = \mathcal{L}^{-1}Y(s)$

$G(s) = \mathcal{L}\,g(t) \rightleftharpoons g(t) = \mathcal{L}^{-1}G(s)$

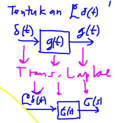
$Y(s) = G(s)X(s)$ ✓

s: peubah LaPlace, peubah KOMPLEKS } frequency domain

Contoh-contoh

* Contoh matematis: * Tentukan $\mathcal{L}\,\delta(t)$

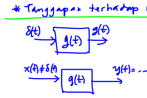
Jawab:



$$G(s) = G(s)\,\mathcal{L}\,\delta(t)$$

$$\mathcal{L}\,\delta(t) = \frac{G(s)}{G(s)} = 1$$

* Tanggapan terhadap isyarat lain



~~$y(t) = \delta(t)$~~

~~$y(t) = g(t)\,x(t) = x(t)\,g(t)$~~

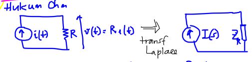
y(t) = konvolusi antara x(t) dan g(t) $\quad \tau$ = tau

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\,g(\tau)\,d\tau = \int_{-\infty}^{\infty} x(\tau)\,g(t-\tau)\,d\tau$$

* Contoh Rangkaian Listrik:

KONSEP IMPEDANSI

Hukum Ohm



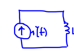
$v(t) = R\,i(t)$

$\mathcal{L}\,v(t) = R\,\mathcal{L}\,i(t)$

$V(s) = R\,I(s)$

$V(s) = Z_R I(s) = R\,I(s)$

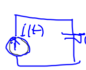
$$Z_R = \frac{V(s)}{I(s)} = R$$



$$v(t) = L\frac{di(t)}{dt} \Rightarrow \mathcal{L}\,v(t) = \mathcal{L}\,L\frac{di(t)}{dt} = L\,\mathcal{L}\frac{di(t)}{dt} = Ls\,I(s)$$

Property # 3, hal 58

$$Z_L = \frac{V(s)}{I(s)} = sL$$



$$i(t) = C\frac{dv(t)}{dt} \Rightarrow \mathcal{L}\,v(t) = \frac{1}{C}\mathcal{L}\int i(t)\,dt$$

$$v(t) = \frac{1}{C}\int i(t)\,dt \qquad V(s) = \frac{1}{C}\frac{I(s)}{s}$$

Property # 4

$$Z_C = \frac{V(s)}{I(s)} = \frac{1}{Cs}$$

* Analisis Transien ✓
* Analisis Keadaan Tunak (Steady State) ✓

* Apa itu isyarat denyut satuan (unit impulse), δ(t)?

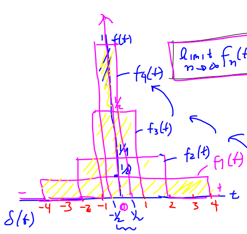
δ(t) adalah suatu isyarat matematis yang tidak bisa di-realisasi secara fisik.

Realisasi fisik yang bisa dianggap mendekati δ(t), antara lain misalnya.

* sambaran petir
* percikan bunga api listrik
* pukulan stick golf pada bola golf
* dll

Yang bersifat "keras" dan "cepat berlalu" dinyatakan dengan 2 sifat matematis:

* $\delta(t)\begin{cases} = 0, & t \ne 0 \\ \ne 0, & t = 0 \end{cases}$ 〉 δ(t) hanya ada pada t = 0
* $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$ 〉 Luas bidang antara δ(t) dengan sumbut adalah satu satuan



$$\lim_{n \to \infty} f_n(t) = \delta(t)$$

* Bagaimana membuat δ(t)?

Ada BANYAK cara membuat δ(t) secara matematis ......

Salah satu cara, adalah sbb..

Next

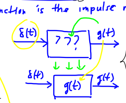
* Model Nisbah-Alih (Transfer Function)

* Apa itu Nisbah Alih?

What is a Transfer Function?

Nisbah-Alih adalah tanggapan denyut

Transfer Function is the impulse response



Nisbah Alih sistem ini adalah g(t) karena ketika diberi masukan isyarat denyut satuan (unit impulse) δ(t), menghasilkan keluaran g(t).

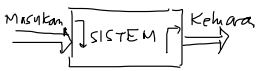
Δ } DELTA

δ } delta

* Pemodelan Ruang Keadaan (State Space Model)

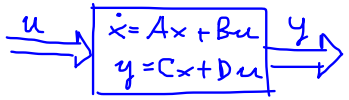
- → MIMO
- → "time domain"
- → MATRIX → Aljabar Linier
  - ↳ SIMULASI komputer
- → "peubah keadaan" ← INTERNAL
  state variables

$\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}$



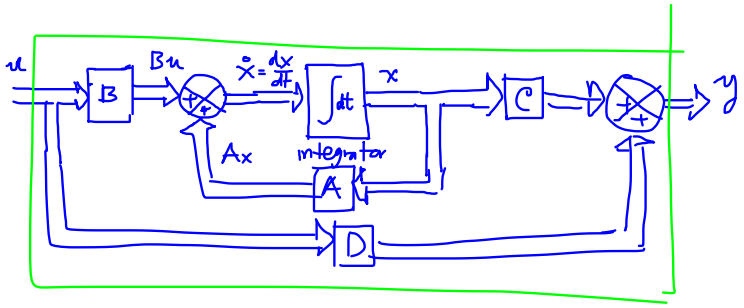
Model Ruang Keadaan

* Pers. Keadaan.
$$\dot{x} = Ax + Bu$$
* Pers. Keluaran
$$y = Cx + Du$$



u = vektor kolom $[m \times 1] = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$. m buah masukan

y = vektor kolom $[k \times 1] = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix}$. k buah keluaran



x = vektor kolom $[n \times 1] = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ peubah keadaan

$\dot{x} = \frac{dx}{dt} = [n \times 1]$

Dimensi A, B, C dan D

$$\underset{[n\times 1]}{\dot{x}} = \underset{[n\times n]}{A}\underset{[n\times 1]}{x} + \underset{[n\times m]}{B}\underset{[m\times 1]}{u}$$

$$\underset{[k\times 1]}{y} = \underset{[k\times n]}{C}\underset{[n\times 1]}{x} + \underset{[k\times m]}{D}\underset{[m\times 1]}{u}$$

A [n x n]
B [n x m]
C [k x n]
D [k x m]

* Tentukan dimensi matrix A, B, C dan D untuk sistem dengan 3 masukan, 4 keluaran dan 5 peubah keadaan !

($m = 3$, $k = 4$, $n = 5$)

Jawab : A [5 x 5], B [5 x 3], C [4 x 5], D [4 x 3]

Catatan :

* Jika semua matrix A, B, C dan D berupa KONSTANTA, maka sistem LTI (Linear Time Invariant)
* Jika ada dr antaranya yang berubah dengan waktu (fungsi t) maka sistem LTV (Linear Time Vary)
* Selain LTI dan LTV → sistem Tak Linier